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INVESTIGATION OF THE PERFORMANCE OF PERSONAL FLOTATION DEVICES

Underwriters Laboratories, Incorporated

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FINAL REPORT

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16. Abstract

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A recommended approach to evaluating PFD effectiveness using experiments with mannequins is described.

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ABSTRACT

An experimental investigation was performed to study various aspects of an existing theory for flotation equilibrium angle of a person wearing a personal flotation device (PFD) in water (Reference 1). The major objectives were determination of the validity of the theory, and derivation of a method for determining the buoyant force and center of buoyancy of a PFD when worn by a person. Additionally, information was obtained on the sensitivity of the theory to small changes in variables, the variability of repetitive measurements of certain human-body characteristics required by the theory (namely, lung vector and intrinsic stiffness vector), the variation with time of day of an individual's intrinsic stiffness vector, and the comparative effectiveness of five PFD's. The experiments used eight human subjects (130-240 lbs. in weight), five PFD's, and five different times of day. Because of the small number of experiments used, the statistical significance of some results is limited.

A recommended approach to evaluating PFD effectiveness using experiments with mannequins is described.

I INTRODUCTION

A previous study (1)* prepared a data base for predicting the performance of personal flotation devices (PFD's) applied to the general population. The study was directed toward providing data on the flotation characteristics of 200 subjects selected from the boating population. A major contribution of this research was to provide experimental data on those human body characteristics that affect buoyancy and flotation stability. A second contribution was the development of a theory and methodology for predicting the performance of PFD's.

Verification of the stability theory and methodology had previously been limited to PFD's consisting of simple foam blocks, for which the centers of buoyancy and buoyant forces could be determined by geometry. These were preliminary tests which compared predicted and observed equilibrium flotation angles. In that work, some discrepancies between theoretical predictions and actual flotation angles were observed.

The overall purpose of the present work is to further investigate the approach proposed in Reference 1 with the objective of eventually using it to evaluate the effects of PFD's on human subjects. It is hoped that thereby conclusions can be drawn about the expected performance

* Numbers in parenthesis refer to the list of references.

of a PFD with respect to the general population, without a need for tests with a large number of human subjects. The current test method requires several human subjects to each don the device, enter the water and follow a specified test procedure to determine what forces/moments the device exerts on the test subject. Subjects must be representative of the three anthropomorphic builds (obese, thin and muscular). Performance of each PFD is compared to the performance of the Coast Guard standard design AK-1 PFD on each test subject. It has been found that at least 10-12 different test subjects must be used to reasonably evaluate performance. This is both time consuming and expensive. Further, tests by human test subjects are not reproducible with a different set of subjects at another time and place.

The present work was directed toward accomplishing the following objectives.

1. Determine the validity of the theory (1) for predicting equilibrium flotation angles for individuals wearing personal flotation devices;

2. Derive a theory and method for calculating a PFD's buoyant force and center of buoyancy when worn by individuals with various anthropometric measurements;

3. Experimentally perform a comparative evaluation of

the effectivenesses of five different PFD's.

4. Gain insight into the magnitude of the variation of an individual's intrinsic stiffness vector during the day, and from one day to another;

5. Determine if a simplified method of estimating the effect of PFD emergence on equilibrium flotation angle is available, and if so, develop it.

Application of the flotation theory of Reference 1 requires the determination of three quantities which are characteristic of the individual and the PFD. These are the intrinsic stiffness vector, the lung vector, and the buoyancy vector. Portions of Reference 1 which outline the derivation of the flotation theory and definition of the above vectors are reproduced in Appendix A. Definitions of the various quantities are given there also.

Objective 1 above requires measurements to determine the three vectors, calculation of equilibrium flotation angles for individuals wearing PFD's, and comparison of the calculated flotation angles with angles observed in flotation experiments. Objective 2 provides the methodology for determining the buoyancy vectors required in the application of the theory. Objective 3 requires flotation experiments with five PFD's to determine

equilibrium flotation angles for various individuals. The experiments used to achieve Objective 1 fill this need. Objective 4 involves measurement of the intrinsic stiffness vectors for a number of individuals on different days and at various times during the day. These determinations are in fact coincident with those used in achieving Objective 1. Finally, Objective 5 involves observing the equilibrium flotation angles and amounts of PFD emergence, and attempting to relate the two in a meaningful way. The required observations are obtained during the flotation experiments for Objective 3.

The procedures required to reach the program objectives therefore involve the following activities:

1. Flotation experiments to determine intrinsic stiffness vectors of a number of individuals at various times.

2. Flotation experiments to determine equilibrium flotation angles and amounts of PFD emergence for the individuals at various times using five different PFD's.

3. Anthropometric and lung-volume measurements for the individuals and calculation of lung vectors from them.

4. Measurements of PFD dimensions when worn by the

individuals, and flotation experiments to determine buoyant forces and centers of buoyancy associated with the PFD's when worn by the individuals.

The methods and equipment described in Reference 1 for determining intrinsic stiffness vector and lung vector were used. Devices and methods for performing the other measurements were developed during the program.

II EXPERIMENTAL AND COMPUTATIONAL METHODS

A. Flotation Test Schedule And Subjects

Eight male subjects were used in the experiments, and these were chosen to represent a reasonable range of physical characteristics. Pertinent individual measurements are given in Table 1.

An attempt was also made to use a mannequin belonging to FAA but furnished through the auspices of the Coast Guard. Unfortunately, the interior of the mannequin had water leaks when received and could not be repaired locally. At the Coast Guard's request the mannequin was returned to the FAA without being used for any testing.

The personal flotation devices used in the tests are shown in Fig. 1. They are the standard USCG AK-1, USCG Design No. 3, Merchant Marine Fibrous Glass, a USCG Type III device, and a hybrid device (combination of air and buoyant material).

Flotation experiments to gather the desired information for the subjects with and without PFD's were performed on eight successive work days. The schedule followed is shown in Table 2. It is seen that Subjects 2-7 were each subjected to five sets of observations on five different days, involving five different times during the day and five different PFD's. Because of illness, Subject 1 could not complete the test schedule, and Subject 1A was used for tests on the sixth and eighth days.

The test sequence for a given subject involved the following steps:

1. Determination Of Lung Volumes Appropriate To The Flotation Theory

As in Reference 1, a spirometer was used to first establish the functional residual capacity (FRC) of the lungs as a reference, and then the change in lung volume from FRC to the condition of the experiment.

Each experiment for measurement of intrinsic stiffness vector was performed with the subject's lungs inflated to produce zero net buoyancy, and the change in lung volume from FRC to zero net buoyancy was determined. Each experiment to determine flotation angle of a subject wearing a PFD was performed with the lungs inflated to an arbitrary comfortable condition, and the change in lung volume from FRC to the arbitrary condition was determined. The methods used were essentially those described in Reference 1.

The lung volume changes and the individual's measurements (Table 1) were used to compute the appropriate lung vectors (LV) as defined in Appendix A.

2. Determination Of The Intrinsic Stiffness Vector

The intrinsic stiffness vector (ISV) was determined by measuring the moments required to maintain the subject at various equilibrium angles under conditions of neutral

buoyancy. This was accomplished by fitting the test subject with a harness, shown in Fig. 2, to which weights and floats were attached so that the combination produced no net buoyancy. Variation of the distance between the floats and weights provided the necessary variation of the moment. The angleometer described in Reference 1 was used to measure equilibrium angle. From the definition of intrinsic stiffness vector (Appendix A), it is seen that the relation between the moment and angle will be sinusoidal for a rigid body. Therefore a sine curve was fitted to the experimental data and from it the magnitude Wd_T and phase angle θ_T of the ISV were found. This was done for both the head back and head forward positions.

This sequence of measurements provided values of the lung vector and intrinsic stiffness vector for each flotation experiment involving one of the PFD's. In addition, repetitive measurements were made of ISV and LV in order to provide an indication of the inherent variability of the measurements.

3. Determination Of Equilibrium Angle And PFD Emergence With The Subject Wearing A Specific PFD

The subject donned the harness and the appropriate PFD, entered the water, and was connected to the angleometer

and spirometer. After the reference lung volume corresponding to FRC was determined, the subject inhaled to a comfortable level and stopped breathing for the remainder of the experiment. The change in lung volume from FRC to the experimental condition was determined from the spirometer measurements. The remainder of the experiment consisted of allowing the subject to reach flotation equilibrium, recording the equilibrium angle, and marking the water line on the partially emerged PFD. This was done for both the head back and head forward positions.

B. Determination Of The Effective Buoyancy And Center Of Buoyancy Of PFD Worn By A Specific Subject

In order to use the flotation theory for prediction of equilibrium flotation angle, it was necessary to determine the buoyancy vector (BV) for the PFD being considered under the exact condition of emergence observed in the flotation experiment. The most direct way of determining center of buoyancy of a submerged body having one plane of symmetry is to determine the torques required to hold the body in two different rotational positions beneath the water surface. For the partially emerged PFD's, this technique is not appropriate for two reasons. Firstly, the PFD cannot be placed in more than one angular position while maintaining

the correct emerged portion; and secondly, changing the rotational position of a PFD would cause it to shift its position and shape unless extraordinay measures were taken to fasten it to a rigid frame. For these reasons, an approximate technique was used to determine center of buoyancy from a single flotation experiment with the PFD mounted on a wire torso frame.

Because of the unusual geometric configurations involved, the computation technique associated with this method is lengthy and difficult to display concisely, even though only elementary geometry and trigonometry are used. For this reason, only the overall concepts involved are explained here, while the details of the computation are given in Appendix C along with a computer program listing.

Figure 3 is a representation of the arrangement used to measure buoyancy and center of buoyancy of a partially emerged device. For this purpose, the PFD is mounted on a wire torso frame which conforms as closely as possible to the torso of the human subject of interest. The weights and centers of gravity of the torso frame and of the PFD are determined beforehand by appropriate measurements in air. These are denoted by CG_T and CG_D, respectively, in the figure. In practice, some adjustment is needed to account for the buoyancies of the emerged and submerged

portions of the torso frame, but those details are omitted here.

The torso frame and device are floated as shown in Fig. 3 with weights W_1 and W_2 adjusted to produce the desired waterline on the device. The effective buoyancy BS of the PFD is then given by

$$BS = W_1 + W_2 + W_T$$

where W_T is the net weight of the frame in the partially submerged position.

The location of the vertical line through CB_S, the center of buoyancy of the device in this position, is determined from the summation of moments about point P, that is:

$$\sum M_P = W_T x_T + W_2 x_W - (BS) x_S = 0$$

$$x_S = (W_T x_T + W_2 x_W)/(BS)$$

Up to this point the determination is exact. An approximation is now introduced in order to determine a second co-ordinate y_S of the center of buoyancy. It is observed first that the center of buoyancy of the fully submerged device CB_D is at or very close to its center of gravity CG_D. If any small difference between CG_D and CB_D is ignored, it can be said that CB_S can neither be above CG_D nor below

the lowest point of the PFD (point L in the example). As an approximation, it is assumed that y_s is proportional to the effective buoyancy of the device BS according to

$$\frac{y_s}{y_d} = \frac{BS}{BD}$$

where BD is the effective buoyancy of the fully submerged PFD. Co-ordinates x_s and y_s can be used to locate CB_s with respect to any desired reference point on the PFD. That information along with measurements made of the subject wearing the PFD allow determination of d_b and θ_b in Figure IV-1 of Appendix A, and hence the buoyancy vector for use in the flotation theory.

III RESULTS AND DISCUSSION

A. Intrinsic Stiffness Vector

The intrinsic stiffness vector, as used in Reference 1, has a magnitude equal to the product of the body weight and the distance between the centers of gravity and buoyancy when fully submerged at zero net buoyancy. The phase angle θ_T is the angle measured from the line joining the CG and CB, to the body axis through the CG (a positive angle is a clockwise angle when observing a right-side profile of the subject). Figure IV-1 of Appendix A illustrates a negative value of θ_T.

Table 3 shows measured values of the magnitude and phase angle of the ISV for the eight subjects employed in the investigation. Data are shown for various times of day for each subject and for three replicate experiments using Subject 3. With these data, a number of questions can be addressed, although the small number of experiments limits the statistical significance of the results.

1. Representativeness Of Sample Of Eight Subjects

In the choice of subjects for this study, an attempt was made to obtain a reasonable range of subject height and weight, but no special sampling techniques

were employed. Table 1 shows that the subjects were in the height and weight ranges of about 5 ft-9 in., to 6 ft-1 in. and 130 to 240 lb and thus encompassed only a segment of the real population. In order to judge the representativeness of the sample for this segment at least, we may compare the means and variabilities of the present data to those from Reference 1. For that purpose, the mean values and standard deviations of the magnitude and phase angle of ISV have been computed for the present data, and for the data of Reference 1 for males over 130 lb in weight. These are shown in the first two lines in Table 4.

Through a modified 2-sided t-test (Reference 7) the following statements can be made concerning the mean values given in Table 4. If $\overline{ISV}_U$ and $\overline{ISV}_A$ designate the true mean values of the magnitude of ISV for the present subjects and for 76 subjects from Reference 1 respectively, then with about 90 percent certainty:

$$-2 < (\overline{ISV}_U - \overline{ISV}_A) < 24 \text{ for the head back position}$$

$$-11 < (\overline{ISV}_U - \overline{ISV}_A) < 15 \text{ for the head forward position}$$

Also, if $\bar{\theta}_{T\ U}$ and $\bar{\theta}_{T,A}$ designate the true mean values of the phase angle of ISV for the present subjects and for 76 subjects from Reference 1 respectively, then with

about 90 percent certainty:

$$-6° < (\bar{\theta}_{T,U} - \bar{\theta}_{T,A}) < 2° \text{ for the head back position}$$

$$-2° < \bar{\theta}_{T,U} - \bar{\theta}_{T,A}) < 9° \text{ for the head forward position}$$

When the 90 percent confidence interval for the difference between two mean values includes zero, as it does in all four cases above, it is customary to state that the true means for the samples involved are equal at a level of significance of 10 percent. Since these confidence intervals are rather small there is reason for believing that the mean magnitudes and phases angles for the present sample of eight subjects correspond rather closely to those for the sample of 76 subjects of Reference 1.

A completely satisfactory method for comparing the variability of the present data with that from Reference 1 is not available because variabilities due to time of day, as well as from person to person, are both involved. Whereas the two variabilities can be estimated for the present date, they cannot be for the Reference 1 data. However, some appreciation can be obtained of the relative variabilities by computing the standard deviation for the present subjects at each specific time of day and comparing these with the data for Reference 1 for all 76 subjects. In order to judge whether or not the present

standard deviations differ significantly from those for Reference 1 subjects, 90 percent confidence intervals are estimated (Reference 3). That is we define S_U and S_A as the respective true values of standard deviation for the present data and Reference 1 data, respectively, and determine the 90 percent confidence interval for S_U/S_A at each time of day. These intervals are given in Table 6.

It may be observed that the confidence intervals are quite wide, thus the comparison of standard deviations is not greatly satisfying in these cases. Nevertheless, the ratio $S_U/S_A = 1$ is included in all but 3 of the confidence intervals; so there is some justification for concluding that the true standard deviations of ISV magnitude and angle for the present subjects do not differ greatly from the corresponding values from Reference 1.

Because of the small number of subjects studied in the present work and the large standard deviations, the above tests of significance necessarily are not very conclusive. However, the results indicate at least that the sample of subjects used is not extremely atypical of the population in the over-130 lb weight class, as represented by the 76 subjects of Reference 1.

2. Variation of Intrinsic Stiffness Vector With Time Of Day

Table 3 indicates that there may be systematic differences of an individual's intrinsic stiffness vector, both in magnitude and phase, at different times of the day. However, interpretation of Table 3 must take account of the fact that there are several sources of variability in the data. These could include at least the following:

a. Random errors of measurement.

b. Inadvertant changes in body position of the subject from experiment to experiment.

c. Changes in the position of the center of lung volume from experiment to experiment, depending on how the chest walls and diaphram, which can move independently, are positioned to establish zero net buoyancy.

d. Changes in weight and density distribution of the body because of changes within the digestive system.

e. Changes in weight and density distribution of the body because of changes in distribution of body fluids.

Items d and e are the day to day variations which are sought, while Items a, b, and c are the inherent variabilities of the experiment. In order to estimate the latter, the data on 3 replicate tests with Subject 3 may be used, although some limitations in the accuracy of the estimate are evident. Firstly, it is probable that the variabilities introduced by Items b and c would be less for a series of consecutive experiments on a single day than for replicate experiments conducted on different days. This statement is based on the observation that a subject would be less and less able to exactly reproduce body position and lung volume with the passage of time. Secondly, since only 3 replicate experiments were possible, the statistical significance of any conclusions cannot be great.

From the replicate tests on Subject 3 (see Table 3), the estimates of standard deviation shown in Table 7 have been made.

Because data from only 3 replicate experiments are available, these cannot be regarded as highly reliable estimates of the true standard deviation. Specifically, it may be observed that the 95 percent confidence intervals for the standard deviations shown in Table 7 are as shown in Table 8 (Reference 4).

Thus it is seen that the confidence intervals are mostly quite large, and therefore it will be possible to identify only very large day-to-day changes in variability with reasonable certainty.

Table 9 shows the standard deviations of ISV magnitude and phase angle for each of six subjects from measurements at five different times of day. It may be observed that the variabilities with time of day appear to be slightly greater than the variabilities observed in replicate experiments of Subject 3. It is of course improbable that the inherent variabilities of ISV measurements of all subjects would be the same as for Subject 3; but in view of the wide confidence intervals on the standard deviations (Table 8), differences between subjects are probably immaterial for present purposes. On that assumption, comparisons are made of the standard deviations for Subjects 2-7 in Table 9 (varying time of day) and for Subject 3 in Table 7 (replicate experiments). Specifically, we define $\hat{S_i}$ and S_r as the true values of standard deviation for Subject i for varying time of day, and for replicate experiments with Subject 3, respectively, and determine whether S_i is significantly greater than S_r. To do this, the value of the ratio $(S_i/S_r)_{90}$ is determined such that there is 90 percent confidence that the true value of $S_i/\hat{S_r}$ exceeds $(S_i/S_r)_{90}$. The results are shown

in Table 10 for six subjects.

When the ratio $S_i/S_r)_{90}$ is less than unity, as it is for most subjects in Table 10, it is customary to state that $\hat{Si}$ is not greater than S_r at a significance level of 10 percent. On this line of reasoning, there is no reason to believe that the day-to-day variability of the ISV data is greater than the variability in replicate experiments, except for the phase angles for subjects 2 and 5 in the head back position. This conclusion must be tempered by the observation that in all cases, the probability that a difference would not be detected at a significance level of 0.10 in these experiments is greater than about 80 percent. This difficulty arises, of course, because the number of experiments performed is too small for good statistical significance.

B. Lung Vector

It may be observed in Figure IV-1 and Eq. IV-1 of Appendix A that the magnitude of the lung vector is the product of the weight of water displaced by a change in lung volume and the distance between the center of the lung volume and the center of the above-water volume of the subject. The locations of the two centers of volume can only be estimated, thus errors of unknown amounts in magnitude and phase angle of the lung vector are introduced. In addition, random errors enter into the

measurement of the change in lung volume. For the flotation experiments with PFD's, the significant change in lung volume is that representing the change from zero net buoyancy to the condition of the experiment. In practice this was determined by evaluating the difference between two lung-volume changes, each of which used functional residual capacity (FRC) as the initial condition. One lung-volume change, that from FRC to zero net buoyancy, was determined during the measurement of ISV prior to a given flotation experiment. The other lung-volume change was that from FRC to the condition used for a flotation experiment with a given PFD. Variability in the measurements of lung vector thus includes inherent experimental errors, as well as any variations of the functional residual capacity of an individual from experiment to experiment.

An estimate of the magnitude of this variability can be made using 12 measurements of the lung-vector magnitude for subject 3 for the change in lung volume from FRC to zero net buoyancy. (See Table 11) The standard deviation estimated from these 12 measurements is 18.0, and the 95 percent confidence interval for the true value is 12.5 to 28.7. If it is assumed that the standard deviation of measurements of the lung vector magnitude for a change in lung volume from FRC to an

experimental condition with a PFD has the same standard deviation, then it may be estimated that the standard deviation of the lung-vector magnitude associated with a change in volume from zero net buoyancy to the condition of an experiment with a PFD is 25.4, and has a 95 percent confidence interval of 17.8 to 40.5. This is tantamount to neglecting errors that may be associated with the determination of the exact point of zero net buoyancy. Thus, the latter standard deviation tends to be overestimated somewhat. It will be shown in a later section that variations of lung vector magnitude by the amount of this standard deviation would have very large effects on the predicted equilibrium flotation angle of a subject PFD combination.

C. Equilibrium Flotation Angle Of Subjects With Various PFD's.

1. Comparison Of Observed And Calculated Equilibrum Angles.

During the flotation experiments with the various PFD's, the equilibrium flotation angles were directly measured. In addition, sufficient information was obtained to determine the magnitudes and phase angles of the intrinsic stiffness vector, the lung vector, and buoyancy vector of the PFD corresponding to each experiment. It was possible therefore, to determine the equilibrium angle predicted by the theory of Reference 1 for each flotation experiment.

A summary of the observed and calculated results is

shown in Table 12, along with the various vector quantities. Each resultant stiffness vector shown there is the vector sum of ISV, LV, and BV, and its angle is identically the predicted equilibrium angle. As shown by the sign convention in Fig. V-1 of Appendix A, positive angles correspond to face-down positions. Comparison of the observed angles with the predicted angles shows extremely poor correspondence in all but a few instances.

Since the flotation theory is formally correct, the source of the discrepancies must be within the experimental data. Although extensive data have not been obtained on the accuracies of the values of the individual vectors shown in Table 12, it is possible to say the following at least.

a. The current best estimates of the standard deviations for repetitive measurements of the magnitude of the ISV of Subject 3 is approximately 11.6 for the head back and 10.6 for the head forward position. These correspond to coefficients of variation* of 9.6 and 8.6 percent, respectively.
b. The current best estimate of the standard deviation for repetitive measurements of the phase angle of the ISV for Subject 3 is 1.3 deg for the head back and 3.3 deg for the head forward position.

---

*Coefficient of variation is the standard deviation expressed as a percentage of the mean value.

c. The current best estimate of the standard deviation for repetitive measurements of the magnitude of the lung vector for Subject 3 is 25.4 in both head positions. This corresponds to a coefficient of variation of 40 percent.

d. Systematic errors of unknown amount are included in both magnitude and phase angle of the lung vectors because the true centers of the lung volumes and the above-water volumes for the subjects are not known.

e. The buoyancy vector is determined by an approximate procedure which introduces uncertainty in the location of the center of buoyancy of the PFD. This, along with uncertainty in the location of the center of the above-water volume, introduces errors of unknown amount in both the magnitude and phase angle of BV.

f. The nature of the experiments suggest that the observed equilibrium angle is reliable within a few degrees and is probably the least source of error in the comparison with predicted angles.

The following section addresses possible implications of these factors.

2. Sensitivity Analysis Of Flotation Theory

In view of the small number of experiments which were possible, the statistical information obtained is scanty, and thus a complete analysis of the propagation of errors is not feasible. Nevertheless, an analysis of the sensitivity of the calculated equilibrium angle to small changes in the input data is possible. Such an analysis can serve two purposes. It can identify parameters to which the prediction is highly sensitive, and thus suggest likely sources of the large discrepancies shown in Table 12. And it can also show the accuracy required in the measurement of the input data for any desired accuracy of the predicted result.

The details of the sensitivity analysis are given in Appendix D. There, the results of the analysis are applied to three subjects from Reference 1 in order to show the consequences of errors in measured quantities. It is perhaps more informative to apply the analysis to

the present subjects, and this has been done for two experiments involving Subject 3, since estimates of the variability of the ISV and LV are available for that subject. The examples chosen represent the smallest and largest values of RSV magnitude computed for Subject 3, and thus encompass the range in sensitivity expected. The sensitivity analysis shows the following:

Subject 3 - Fibrous Glass Device - Head Back (RSV Magnitude = 21)

Error in θ for 1° error in θ_T - 0.79°

Error in θ for 1° error in θ_L - 0.03°

Error in θ for 1° error in θ_B - 0.15°

Error in θ for 1% error in I - 3.18°

Error in θ for 1% error in L - 1.02°

Error in θ for 1% error in β - 4.74°

Subject 3 - Design 3 - Head Back (RSV Magnitude = 142)

Error in θ for 1° error in θ_T - 0.58°

Error in θ for 1° error in θ_L - 0.37°

Error in θ for 1° error in θ_B - 1.95°

Error in θ for 1% error in I - 0.27°

Error in θ for 1% error in L - 0.12°

Error in θ for 1% error in β - 0.39°

Here the quantities I, L and $-\beta$ are the magnitudes of the intrinsic stiffness vector, the lung vector, and the buoyancy vector, respectively.

From these values, the potential errors corresponding to estimated standard deviations of measured quantities may be evaluated as shown in Table 13.

There is no information at present on the possible errors of measurement of the magnitude and angle of the buoyancy vector or the angle of the lung vector. It is reasonable to suppose, however, that errors of 10 percent in the magnitude and 5° in the angles could easily occur. These are used in the example shown in Table 14.

As pointed out in Appendix A, the sensitivity analysis is numerically correct only for very small increments in the variables. Thus, the large increments in some variables in Tables 13 and 14 must be regarded only as indicative of potential sources of large error.

The tables show that the largest sensitivity to errors occurs for the fibrous glass device where the calculated magnitude of the resultant stiffness vector (RSV) is 21. For that device, random errors in ISV and LV along with a small error in BV could well be responsible for the large difference (70°) between calculated and observed equilibrium angles. Analysis of other situations wherein the magnitude of RSV is small would undoubtedly lead to the same conclusion.

For the Design 3 device, Tables 13 and 14 indicate that random errors in ISV and LV magnitudes are unlikely to

cause large errors in θ. For example, if ISV and LV magnitudes were in error by two standard deviations simultaneously to produce additive errors, the error in θ would be only about 15°. Yet the probability that either error would be two standard deviations or more is less than 0.05 (for normally distributed errors) and the probability that both would have that magnitude simultaneously is probably considerably less than .01. Furthermore, the tables show that very large errors in magnitude and angle of BV or the angle of LV would be required to produce the observed error in predicted equilibrium angles (110° in this case). Analysis of other situations wherein the magnitude of RSV is large would undoubtedly lead to a similar conclusion.

Examination of Table 12 shows that there is no apparent correlation between the magnitude of RSV and the discrepancy between observed and calculated equilibrium angles. This strongly suggests that random errors in ISV and LV are not entirely responsible and that large unidentified errors are present in some of the data. It is pointed out that the true magnitude of RSV is not known for any of the experiments. Since the predicted angle of RSV is not reliable, the predicted magnitude must be viewed with suspicion as well.

Although the limited scope of this study does not permit further investigation to identify the errors, it appears that the most likely sources are the approximations used in locating the center of lung volume, center of above-water volume of the subject, and the center of buoyancy of the PFD. The potential for error is probably greatest in the latter, since the buoyancy vector will usually have a larger effect on equilibrium angle than the lung vector. This thesis is supported by the observation that large values of buoyancy vector magnitude seem to be associated with large errors in predicted angle in Table 12.

The determination of the buoyancy vector for a given device is an inherently inexact procedure because of the practical difficulties associated with available experimental techniques. The determination involves first a flotation experiment with a subject, and second a flotation experiment with the PFD mounted on a wire frame which is supposed to duplicate the shape of the subject's torso. A reference point on the PFD must be accurately located with respect to reference points on the subject and on the frame.

In the flotation experiment with the frame, the emerged portion of the PFD must duplicate that which occurred with the subject, and the buoyant force developed must be measured. In addition, the points of application

of downward forces on the wire frame must be located with respect to the chosen reference point on the PFD. From the measurements with the PFD on the wire frame, the location of the center of buoyancy of the PFD relative to the reference point is then estimated by the approximate procedure described previously. Finally, the location of the center of buoyancy of the PFD relative to the center of the above-water volume of the subject is calculated from the above information and from physical measurements of the subject.

It is evident that there are numerous sources of error in the above procedures. These arise mainly because of the practical difficulties involved with measuring distances between the various parts of the subject and the PFD, both of which are non-rigid bodies, and of duplicating with the wire frame the exact position of the PFD as worn by the subject. Errors are also introduced by the approximations used in locating the center of the above-water volume of the subject and the center of buoyancy of the PFD. The comparisons of observed and predicted equilibrium angles and the sensitivity analysis suggest that the accumulation of errors involved in these procedures can be so large that no confidence can be placed in predicted angles.

As a side issue, it may be observed that the magnitude of RSV is identically equal to $\partial M/\partial \theta$ at equilibrium. Thus it is indicative of the stability of the equilibrium position (M is net turning moment on the subject). It is clear therefore that a small change in any turning moment could have a large effect on the equilibrium angle whenever RSV has a small magnitude.

Although the foregoing analysis involves sensitivity of a theory to changes in variables, the sensitivities estimated apply to the physical behavior of a subject - PFD system as well. Thus, a condition which leads to highly stable flotation of a subject is one in which the magnitude of RSV is large. This raises the question as to the possible use of RSV magnitude in the evaluation of the effectiveness of a PFD.

D. Comparative Effectiveness of PFD's.

A number of factors would contribute to the ability of a device to hold a subject in a face-up position (negative equilibrium angle) and resist forward turning. Among these are; large buoyant force, CB located far from the head, and CB located far in front of the body axis. In the terminology of the flotation theory, the desired properties are a buoyancy vector with large magnitude and positive phase angle. In addition, it is desirable to have a resultant stiffness vector with large magnitude

in order to assure stability of the equilibrium position. In principle then, it should be possible to compare the various PFD's on the basis of buoyancy vector and resultant stiffness vector. However, for the present study, the computed angle of RSV has been shown to be unreliable for estimating equilibrium flotation angle, and thus the computed magnitude of RSV must be considered questionable as well.

Table 15 shows the observed equilibrium flotation angles and Table 16 shows the buoyancy vectors determined for the various PFD - Subject combinations. Comparisons between buoyancy vectors and flotation angles shows a number of apparent inconsistencies, where negative angles of the buoyancy vector occur with negative flotation angles (2F with Fibrous Glass, 3F with Design 3 and Fibrous Glass, 6F with AK-1); or where positive angles of the buoyancy vector occur with positive flotation angles (2F with Type III, 3F with Type III). This further suggests that the buoyancy vectors determined by the approximate procedure described previously are not accurate enough to be useful in predicting flotation angle of a subject.

The observed equilibrium flotation angles are useful for judging the relative effectiveness of the various

PFD's (Table 15). Since a negative angle corresponds to the face-up position, this is the desirable condition. It is seen that the Design 3, Fibrous Glass, and Hybrid devices floated all subjects face up at least 20° back from vertical; and the AK-1 did the same except for Subject 1 in the head forward position. In that case however, the Subject was floated high enough that the slight forward angle was acceptable. On the basis of this limited data, it can be said that these four devices appear to be similar in effectiveness, with a slight reservation with respect to the AK-1. The Type III device, on the other hand, was able to float only 1 of the 6 subjects at a negative (face up) angle in the head-forward position, and then at only -1°. This appears to be a lower level of performance than that shown by the other devices.

Consideration of the physical characteristics of the devices (see Fig. 1) shows why the devices behave as they do. Devices which have centers of buoyancy far from the head, will also tend to produce large buoyant forces because the subjects are raised far out of the water. Both factors lead to a large magnitude of BV. Design 3 has these characteristics and in addition its center of buoyancy is well in front of the body axis. It is not surprising then that it is one of the most effective devices. The

AK-1, Fibrous Glass, and Hybrid Devices would apparently not raise the subjects as high above the water as would the Design 3 device, and thus would produce somewhat lesser buoyant forces. Nevertheless, all have centers of buoyancy well below the head and in front of the chest, and thus would be expected to perform well. The Type III device, however, appears to have its center of buoyancy near the body axis. For this reason, it is capable itself of producing a forward turning moment for the head forward position, and cannot rotate a subject out of a face-down position, even though it may have a large buoyant force and a center of buoyancy well below the head.

E. Effect Of PFD Emergence On Equilibrium Flotation Angle.

The effect of PFD emergence on the flotation angle of a subject involves its effect on both the buoyant force B and the location of the center of buoyancy CB. The relationship of the buoyant force to the emergence is direct, whereas the relationship of the location of center of buoyancy to emergence is quite complex. As described previously, an approximate method was used to estimate the location of CB for partially emerged devices. Even so, rather involved computations were necessary, so it would be desirable to have a simpler approach. For the present, this seems to be infeasible since the approximate method

used here does not itself appear to be sufficiently accurate for predicting CB, and a simplified procedure is not likely to offer any improvement. The following discussion illustrates the complexity of the situation.

Depicted schematically in Fig. 4 is a partially emerged PFD positioned in a manner typical of a head-back equilibrium. In assessing the effect of emergence on the equilibrium angle, we observe the following. The turning moment caused by the PFD is the horizontal component of the buoyancy vector which is numerically equal to $-Bd_B \sin(\theta-\theta_B)$. (In the sign convention used, a positive moment tends to produce rotation through a negative angle, that is, counterclockwise in Fig. 4). A change in the amount of emerged material tends to change all of the quantities B, d_B, and θ_B, which could produce opposing effects on the turning moment. For example, if the emergence is increased with θ held constant, the buoyant force B decreases tending to reduce the moment. On the other hand, as the emergence increases, the point CB moves farther away from CBD along the line through CBE and CBD, and CVb moves closer to the water line. Note that this corresponds to movement of the PFD upward relative to the subject. With a given subject, this must occur because the above-water volume of the subject must decrease as the buoyant force decreases.

Depending upon the configuration of the system, these movements of CB and CV_o could increase or decrease d_B and θ_B and thus could tend to increase or decrease the turning moment. Movement of CV_o would also affect the lung vector to some degree.

For certain PFD's, the largest effect would be produced by the change in B, and increasing emergence would result in decreasing buoyancy vector. The result would be forward turning of the subject until the balance between net stiffness vector and buoyancy vector were re-established at an angle θ nearer to the vertical. This situation appears to apply to devices with emerged material in front of the subject's chest, such as the Design 3 and Type III Devices. For PFD's with significant amount of buoyant material behind the head, such as the AK-1 and Hybrid Devices, the predominant effect would probably be the increase in d_B with increasing emergence, resulting in increasing buoyancy vector and rearward turning of the subject.

Generalization of the above reasoning to comparisons of different subjects is obviously much more complicated since the differences in net stiffness vector and location of the center of the above-water volume will have large effects. For this reason, the use of the present data to show the effect of PFD emergence is rather inconclusive.

Table 17 shows the emergence of the various PFD's (in percent of total buoyancy utilized) along with the equilibrium angles for the eight subjects. Only for the Fibrous Glass Device does a trend appear to be present. For that device, the equilibrium position seems to move away from the vertical as emergence of the PFD increases. This would place the Fibrous Glass device in the category for which loss in buoyancy behind the head results in an increasing rearward turning moment. The configuration of the device is compatible with this characteristic, but the variability among subjects may well be responsible for the trend shown. For the other devices used, it appears that variations among individual subjects mask any relationship that might exist between emergence and equilibrium angle.

IV CONCLUSIONS

1. A review of the theory developed in Reference 1 for flotation of individuals wearing personal flotation devices (PFD's) in water indicates that it is formally correct in that all forces and moments are accounted for, and that the equilibrium condition imposed corresponds to a condition of zero net turning moment. Several practical difficulties in the application of the theory are apparent, however. These arise from the requirements that the subject - PFD system be considered rigid and that the centers of volume of the lungs and above-water parts of the subject and the center of buoyancy of the PFD be known. In addition there is the question of the reproducibility of measurements of intrinsic stiffness vector (ISV) and lung vector (LV) of an individual, and the question of the extent of variation of LSV with time.

2. Replicate flotation experiments with Subject 3 showed that the estimated coefficients of variation for the magnitude of ISV are about 9.6 and 8.6 percent for the head back and head forward positions, respectively. Also, the estimated standard deviations for the phase angle are 1.3 and 3.3 degrees for the same respective head positions (Table 7), However, the 95 percent confidence intervals on these are quite large because only three experiments were performed (Table 8).

3. Estimated coefficient of variation of the lung vector magnitude for both head positions measured in 12 replicate experiments with Subject 3 was about 29 percent with a 95 percent confidence interval of about 20 to 46 percent. For the conditions of flotation experiments for this subject with a PFD, the coefficient of variation would be larger for reasons given in Section III-B, amounting to about 40 percent with a 95 percent confidence interval of 28 to 65 percent.

4. The mean values and standard deviations of ISV magnitude and phase angle for eight subjects used in the experiments were compared with corresponding quantities derived for 76 male subjects in the over 130-lb. weight class (Reference 1). Statistical tests indicate no reason to believe that these quantities differ significantly for the two sample populations at a level of significance of 0.10. Because of the small number of subjects in the present work, and the large standard deviations involved, the statistical tests of standard deviations are not very conclusive. Nevertheless, the tests provide some assurance that the sample of subjects used is not extremely atypical of the general population in the corresponding weight class.

5. Variability of the ISV with time of day was determined for six subjects and five different times. The standard deviations of ISV magnitude and phase angle for the six subjects with varying time are mostly larger than the standard deviations measured for replicate experiments with Subject 3. Statistically however, the variabilities with time of day are not larger than the variability of the measurement for Subject 3 at a significance level of 0.10, except for phase angles for two subjects. The experiments thus have not conclusively identified a variability of ISV with time. This conclusion must be tempered by the observation that because of the small number of experiments, the probability detecting a difference at this level of significance is quite small (about 20 percent).

6. The experimental determination of the buoyancy vector for specific subject - PFD combinations is at best an approximate procedure beset with a number of practical difficulties. These arise mainly from the problems involved with measuring distances between the various parts of the subject and the PFD, both of which are not rigid, and of duplicating with a wire frame the exact position of the PFD as worn by the subject. Inaccuracies are also introduced because only approximate

procedures are available for determining the locations of the center of the above-water volume of the subject and the center of buoyancy of the device, even if the various distances were measured without any error.

7. When equilibrium flotation angles are computed from measured values of ISV, LV, and BV, the correspondence with observed flotation angles is extremely poor with differences of over 100 degrees being common (Table 12). In only 6 of 38 cases were the differences in angles less than 20 degrees, and in only 11 cases were the differences less than 45 degrees.

8. A sensitivity analysis of the theory showed that when the resultant stiffness vector (RSV) has a small magnitude, the inherent variability of the measurements of ISV and LV and small experimental errors in BV could be responsible for the observed lack of agreement between calculated and observed equilibrium angles (Tables 13 and 14). When the magnitude of RSV is large however, very large relative errors in the measured quantities would be required to cause the existing differences between the calculated and observed equilibrium angles. Since there seems to be no correlation between the magnitude of the resultant stiffness vector and the inaccuracy of the predicted angle, it is probable that large relative errors in some measured quantities,

as well as inherent variabilities in ISV and LV, are responsible. It must be pointed out, however, that the true magnitude of RSV is not known for any of the experiments. It is concluded therefore that inherent inaccuracies in the measurements used in this work for the various vector quantities are too great to permit useful predictions of equilibrium angle using the flotation theory of Reference 1. The nature of the experiments suggests that the procedure for determining buoyancy vector is the most probable source of large errors. However, in certain cases, the assumption that the center of volume of the above-water part of the subject is located at the external meatus (ear) may introduce large errors in both LV and BV, and these may be additive.

9. Because of the inherent inaccuracy of the measurement of center of buoyancy of partially emerged PFD's, and the extreme sensitivity of the flotation theory to small experimental errors in some cases, direct measurement of center of buoyancy does not appear to be a worthwhile approach to evaluation of PFD effectiveness.

10. The limited comparisons possible in the investigation show the AK-1, Design 3, Fibrous Glass, and Hybrid Devices to be approximately equal in effectiveness to float the subjects at large negative angles, that is

at angles well back from the vertical (Table 15). The Type III device, on the other hand, could not maintain negative flotation angles for 5 of 6 subjects tested in the head forward position. No general method, simplified or otherwise, was found for estimating the effect of PFD emergence on equilibrium flotation angle.

11. The sensitivity analysis shows that the magnitude of the resultant stiffness vector is a measure of the moment necessary to turn a subject away from the equilibrium position. As such, it may be useful in specifying PFD effectiveness.

V RECOMMENDATIONS

It is recommended that an approach to evaluation of PFD effectiveness be developed which does not rely on measurements to determine center of buoyancy of the PFD. An approach of this kind appears to be entirely feasible, providing only that sufficient information can be obtained on the physical characteristics of the population. Such information may, in fact, already be available within Reference 1.

The approach recommended here can be explained using Figure 5, which is a polar representation of the various vector quantities of the type used in Reference 1. Shown there is the graphical subtraction of a buoyancy vector BV from a net stiffness vector NSV to produce a resultant stiffness vector RSV. It will be recalled that NSV is the sum of the intrinsic stiffness vector and the lung vector produced by the lung volume change from zero net buoyancy to functional residual capacity. The angle of RSV corresponds to the flotation angle measured from the vertical to the body axis. A negative angle represents backward rotation of the individual.

One could postulate a range of equilibrium angles required to float a subject with his face out of water. As an example we could use the range from 0 to (-)90°.

This requires that the RSV always be in the upper left quadrant of Figure 5. It is therefore desired to know what limitations on the buoyancy vector will guarantee this condition for a specified population. The population characteristic of interest is the range of NSV involved.

The data presented in Reference 1 show that the domain of NSV for a specified population can be represented by a region similar to the sector enclosed by FBB'CGF'F in Figure 5. We may therefore determine the limitations on BV such that any NSV within the chosen sector results in an RSV within the upper left quadrant. It happens that this determination can be made by considering only the corners F, G, B, and C of the sector.

It can be shown that when the NSV is coincident with OB, the RSV will be in the upper left quadrant whenever the terminus of BV falls in the region to the right of the line AB and below the line BD. Similarly, for NSV coincident with OC, the corresponding limits on BV are described by KC and CD. For NSV coincident with OF, the limits on BV are described by EF and FI; and for NSV coincident with OG, the limits on BV are described by JG and GI. It may now be observed that whenever the terminus of BV falls within the shaded region to the right of and below KHI, all individuals with NSV within the region FBB'CGF'F will be floated at an angle between 0 and -90°.

As mentioned, it is not considered feasible to determine BV, therefore an indirect approach is required. This approach can rely on the fact that if individuals with NSV's equal to OG and OC are both floated at angles between 0 and -90°, then the terminus of BV must be in the shaded region, and the desired condition on θ will be satisifed for the population of interest.

In principle then, it is possible by this method to evaluate PFD effectiveness for a given population using only two tests with individuals with specified net stiffness vectors. In practice, however, the procedure is complicated by the effects of body size and weight, and the variation of buoyancy vector with flotation angle. These problems are not insurmountable provided that sufficient experimentation can be done over the range of body characteristics of interest to determine which limiting experiments should be performed to take account of these extraneous effects. It appears that independent variations of body size, weight, and NSV will be required for these experiments.

In view of the latter statement, it is recommended that the feasibility of developing a mannequin (or several mannequins) which can embody the desired range of variables be investigated. The data of Reference 1 on body characteristics should be used to the fullest extent possible in

this work. Assuming a favorable outcome, a prototype mannequin should be developed and pilot experiments carried out with it. Ideally, the mannequin should be designed for independent variation of NSV, weight, and physical size of the torso area, but more than one mannequin may be required to accomplish these goals. It may be observed that the mannequin need not simulate the human form except as it affects the fitting of the PFD. A simple frame with movable weights and floats and a PFD mounting frame would seem to be entirely adequate.

Experiments with the mannequin(s) and human subjects will indicate the correct approach and its reliability. At some point within this sequence of development it may be necessary to obtain more information on human body characteristics and/or to investigate the confidence limits for the NSV data available in Reference 1.

This approach holds promise of providing a means to evaluate equilibrium flotation angle using a few well-defined experiments without the need for human subjects. It may also be possible to perform other evaluations by a similar technique, such as determining the ability of a PFD to turn an individual from a face-down to a face-up position. Although it does not appear possible, nor desirable, to accurately measure the magnitude of the resultant

stiffness vector, it may be worthwhile to determine stability of the equilibrium position by measuring the rate of change of righting moment with angular displacement.

TABLE 1

PHYSICAL CHARACTERISTICS OF SUBJECTS

	SUBJECT NUMBER							
	1	1A	2	3	4	5	6	7
Height (Ft-in.)	5 $9\frac{13}{16}$	5 $9\frac{1}{8}$	5 $10\frac{5}{16}$	5 10	6 $1\frac{1}{8}$	5 $11\frac{11}{16}$	5 10	5 $9\frac{1}{2}$
Weight (lbs.)	$159\frac{1}{2}$	$214\frac{1}{2}$	238	$184\frac{1}{2}$	$167\frac{1}{2}$	161	$174\frac{1}{2}$	134
Age (years)	19	22	28	27	49	45	30	-
Chest Thickness (in.)	$9\frac{1}{2}$	12	$11\frac{7}{8}$	$10\frac{1}{2}$	10	9	9.7	$9\frac{3}{8}$
Distance from Chest Front to SSN (in.)	$2\frac{1}{2}$	$4\frac{5}{8}$	$4\frac{1}{2}$	$3\frac{3}{8}$	$3\frac{7}{8}$	$3\frac{1}{16}$	3.4	$2\frac{5}{16}$
Chest Front to CV_L (inches)	$5\frac{1}{8}$	$6\frac{1}{2}$	6	$5\frac{1}{2}$	$5\frac{7}{8}$	$4\frac{1}{2}$	5.6	$4\frac{7}{8}$
Vert. Distance SSN to CV_L (inches)	$7\frac{1}{2}$	7	$8\frac{1}{2}$	$8\frac{1}{2}$	$7\frac{3}{4}$	$7\frac{7}{8}$	7	$8\frac{3}{8}$
CV_L to CV_o (head forward (inches)	$15\frac{1}{2}$	14	15	$14\frac{1}{2}$	$15\frac{1}{2}$	$14\frac{1}{2}$	15	15
*Angle from Vertical (head forward) (deg.)	-28	-27	$-19\frac{1}{2}$	-16	-17	-30	-24	-22
CV_L to CV_o (head back (inches)	15	14	$14\frac{1}{2}$	$14\frac{1}{2}$	$14\frac{3}{4}$	14	$15\frac{1}{2}$	$15\frac{1}{2}$
*Angle from Vertical (head back) (degrees)	-2	6	4	-1	17	8	18	4

*Subject standing and body axis assumed to be vertical when measurements taken. Therefore, this angle equals θ_L, the angle between the line from CV_L to CV_o and the body axis.

TABLE 2

TEST SCHEDULE

DAY	SUBJECT NUMBER 1	1A*	2	3	4	5	6	7
1	AL^1	---	BS^2	---	---	BB^1	AB^1	BL^1
2	---	---	---	BB^2	AB^2	AL^2	BS^2	---
3	AB^3	---	BL^3	AL^3	BS^3	---	---	BB^2
4	BS^4	---	---	---	---	AB^3	BL^3	AL^3
5	---	---	BB^4	AB^4	BL^4	BS^4	---	---
6	___	BL^5	AL^5	BS^5	---	---	BB^4	AB^4
7	---	---	---	---	BB^5	BL^5	AL^5	BS^5
8	---	BB^2	AB^1	BL^1	AL^1	---	---	---

BB=Before Breakfast
AB=After Breakfast
BL=Before Lunch
AL=After Lunch
BS=Before Supper

AK-1 = 1
Design 3 = 2
Type III = 3
Fibrous Glass = 4
Hybrid = 5

*A substitute subject used here due to illness of Subject #1.

TABLE 3

INTRINSIC STIFFNESS VECTOR (ISV)

This table lists ISV magnitudes in inch-pounds and angles as calculated from observed data taken for each subject just prior to testing while wearing a PFD. See Table I for test Schedule.

Head Position	Time of Day	SUBJECT NUMBER 1	1A	2	3	4	5	6	7
Back	BB	--	77.37	77.99	106.78	104.57	146.03	78.47	100.2
		--	-13.09°	-4.80°	-11.35°	-5.11°	-30.07°	-8.71°	-3.10°
Forward	BB	--	57.51	81.69	126.73	116.97	49.94	93.48	103.0
		--	15.4°	7.63°	-4.25°	-0.55°	-3.62°	-0.96°	16.67°
Back	AB	45.73	--	79.97	117.78	118.17	98.25	76.04	100.92
		-1.86°	--	-5.31°	-8.06°	-4.48°	-9.06°	-6.51°	-8.86°
Forward	AB	70.91	--	78.86	100.59	149.06	104.7	109.21	107.91
		10.36°	--	7.40°	1.39°	-6.70°	-4.77°	4.76°	16.61°
Back	BL	--	100.1	68.54	122.02	95.02	108.69	97.23	104.24
		--	-14.52°	-4.61°	-15.76°	-12.09°	-10.28°	-14.02°	-8.49°
Forward	BL	--	82.85	58.96	118.79	79.65	101.92	100.74	90.62
		--	15.01°	0.19°	-7.77°	-3.54°	-5.62°	-4.84°	12.06°
Back	AL	93.09	--	87.58	143.45	110.42	97.13	106.44	98.36
		-14.2°	--	-6.04°	-16.54°	-10.55°	-20.86°	-7.74°	-2.96°
Forward	AL	87.4	--	94.06	99.1	108.63	102.63	114.02	108.82
		7.54°	--	-1.75°	-1.09°	-0.20°	-9.72°	-3.15°	15.59°
Back	BS	116.72	--	84.04	132.09	83.47	87.39	92.79	114.83
		-16.56°	--	6.02°	-12.04°	-8.43°	-16.46°	-13.10°	-2.04°
Forward	BS	120.09	--	87.72	125.55	103.4	88.80	88.96	124.03
		-4.03°	--	10.14°	-4.68°	-2.57°	-11.43°	1.56°	12.61°

REPLICATE TESTS OF #3 DURING ONE SESSION (AB)

Data Taker	#1	#2	#1
Head Back	115.91	113.42	134.66
	-13.6°	-16.03°	-15.71°
Head Forward	111.45	131.99	126.44
	-5.58°	-10.71°	-4.62°

Time of Day:
- BB=Before Breakfast
- AB=After Breakfast
- BL=Before Lunch
- AL=After Lunch
- BS=Before Supper

TABLE 4

COMPARISON OF INTRINSIC STIFFNESS VECTORS

This table shows for comparative purposes the mean values and estimated standard deviations of ISV magnitudes (in-lbs) and angles obtained for each test subject. The averages for 76 males weighing over 130 pounds from Reference 1 are also shown.

SUBJECTS	HEAD BACK Magnitude	HEAD BACK Angle, Degrees	HEAD FORWARD Magnitude	HEAD FORWARD Angle, Degrees
ADL (76 males	88.1 (24.5)	-11.6 (9.4)	100.3 (25.9)	6.72 (11.6)
8 UL & USCG SUBJECTS	99.2 (20.3)	- 9.76 (6.07)	95.21(20.93)	2.11 (8.27)
#1	85.2 (36.2)	-10.9 (7.9)	97.0 (24.7)	7.5 (10.5)
#1A (2 tests)	88.7 (16.7)	-13.8 (1.0)	70.2 (17.9)	15.2 (.28)
#2	79.7 (7.2)	-2.9 (5.0)	80.3 (13.3)	4.7 (5.2)
#3	124.4 (14.0)	-12.75(3.46)	114.0 (13.4)	-3.3 (3.5)
#3 - Repeats	121.3 (11.6)	-15.1 (1.3)	123.0 (10.6)	-6.97(3.3)
#4	102.3 (13.5)	-8.1 (3.3)	111.5 (25.1)	-2.7 (2.6)
#5	107.5 (22.8)	-17.3 (8.6)	89.6 (23.0)	-7.0 (3.4)
#6	90.2 (12.8)	-10.2 (3.2)	101.3 (10.5)	-.53(3.8)
#7	103.71 (6.57)	-5.1 (3.3)	106.9 (12.0)	14.7 (2.2)

*Estimated Standard Deviations are shown in parentheses.

TABLE 5

STANDARD DEVIATIONS OF ISV
MAGNITUDES AND PHASE ANGLES FOR EIGHT
SUBJECTS AT VARIOUS TIMES OF DAY
AND FOR 76 SUBJECTS FROM REFERENCE 1

Eight Present Subjects

Time of Day	Head Back Magnitude	Head Back Angle	Head Forward Magnitude	Head Forward Angle
BB	24.6	9.21	28.8	8.89
AB	25.8	2.62	25.2	8.26
BL	16.3	3.91	19.2	9.08
AL	18.5	6.29	9.31	8.18
BS	19.4	8.30	17.5	8.56

76 Subjects (Reference 1)

Unknown	24.5	9.4	25.9	11.6

TABLE 6

90 PERCENT CONFIDENCE INTERVALS
FOR THE RATIO OF STANDARD DEVIATIONS OF
PRESENT DATA AND REFERENCE 1 DATA FOR ISV

Quantity	Head Position	Time of Day	90% Confidence Interval For S_U/S_A
Magnitude	Back	BB	0.68 to 1.94
"	"	AB	0.71 to 2.02
"	"	BL	0.45 to 1.29
"	"	AL	0.52 to 1.46
"	"	BS	0.53 to 1.52
"	Forward	BB	0.75 to 2.12
"	"	AB	0.65 to 1.86
"	"	BL	0.50 to 1.42
"	"	AL	0.24 to 0.68
"	"	BS	0.46 to 1.30
Angle	Back	BB	0.66 to 1.88
"	"	AB	0.19 to 0.53
"	"	BL	0.28 to 0.80
"	"	AL	0.45 to 1.29
"	"	BS	0.59 to 1.69
"	Forward	BB	0.52 to 1.46
"	"	AB	0.48 to 1.36
"	"	BL	0.52 to 1.48
"	"	AL	0.48 to 1.36
"	"	BS	0.50 to 1.42

TABLE 7

STANDARD DEVIATIONS FOR THE ISV OF SUBJECT 3 IN THREE REPLICATE EXPERIMENTS

Head Position	Standard Deviation Magnitude	Phase Angle
Back	11.6	1.32
Forward	10.6	3.27

TABLE 8

95% CONFIDENCE INTERVALS FOR THE STANDARD DEVIATION OF REPLICATE MEASUREMENTS OF THE ISV OF SUBJECT 3

Head Position	95% Confidence Interval Magnitude	Phase Angle
Back	5.31 to 56.4	0.60 to 6.42
Forward	4.85 to 51.2	1.50 to 15.89

TABLE 9

STANDARD DEVIATIONS OF THE
MAGNITUDE AND PHASE ANGLE OF ISV FOR EACH
SUBJECT OVER 5 DIFFERENT TIMES OF DAY

Subject	Head Position	Standard Deviation Magnitude	Standard Deviation Angle
2	Back	7.22	5.04
2	Forward	13.3	5.18
3	Back	14.0	3.46
3	Forward	13.4	3.52
4	Back	13.5	3.32
4	Forward	25.4	2.63
5	Back	22.8	8.57
5	Forward	23.0	3.37
6	Back	12.8	3.34
6	Forward	10.5	3.81
7	Back	6.6	3.30
7	Forward	12.0	2.22

TABLE 10

LOWER 90 PERCENT CONFIDENCE
LIMITS ON THE RATIO S_i/S_r FOR SIX SUBJECTS

Subject	Head Position	(S_i/S_r) Minimum Magnitude	(S_i/S_r) Minimum Angle
2	Back	0.20	1.25
2	Forward	0.43	0.53
3	Back	0.39	0.86
3	Forward	0.42	0.35
4	Back	0.38	0.80
4	Forward	0.79	0.33
5	Back	0.64	2.14
5	Forward	0.71	0.34
6	Back	0.36	0.83
6	Forward	0.33	0.38
7	Back	0.19	0.82
7	Forward	0.34	0.22

TABLE 11

VARIATION IN LUNG VECTOR FOR TEST SUBJECT NO. 3

Date	Subject & Time*	Head Forward d_L (inches)	Head Forward θ_L (deg.)	Head Back d_L (inches)	Head Back θ_L (deg.)	ΔV_L (liters)	Lung Vector (LV) Magnitude
10/3	3BB	14.5	-16	14.5	-1	2.74	87.6 in-lbs
10/3	(Repeat)					1.35	43.2
10/8	3AB					1.86	59.5
10/8	(Repeat)					1.21	38.4
10/11	3BL					1.2	38.4
10/4	3AL					2.73	87.3
10/9	3BS					1.93	61.7
10/15*	3AB					2.1	67.1
10/15*	3AB					2.3	73.5
10/15*	3AB					1.52	48.6
10/15*	3AB					2.15	68.7
10/15*	3AB					2.58	82.5
	LV Mean =						63.01
	Std Dev =						18.0

ΔV_L - Change in lung volume from that yielding zero net buoyancy in fresh water to functional residual capacity. For Subject No. 3, the change was always a decrease in volume (positive change).

d_L - Measured distance in inches between the subject's center of lung volume and center of ear. For Subject No. 3 d_L happened to be the same for both head-forward and head-back positions.

$LV = P_W \Delta V_L d_L$ where $P_W = 2.2046$ lbs/litre

*NOTE: Five consecutive measurements taken during one After Breakfast session. Mean LV for these was 68.08 with a Standard Deviation of 12.4.

TABLE 12

SUMMARY OF CALCULATED AND OBSERVED RESULTS

Subj(1)	PFD(2)	Lung Vector Mag. (In-lbs)	Lung Vector Angle (Degrees)	Intrinsic Stiffness Vector Mag. (In-lbs)	Intrinsic Stiffness Vector Angle (Degrees)	Buoyancy Vector Mag. (In-lbs)	Buoyancy Vector Angle (Degrees)	Resultant Stiffness Vector Mag. (In-lbs)	Resultant Stiffness Vector Angle (Degrees)	Observed Equilibrium Angle (Degrees)
1-B	AK-1	- 14.2	- 2	93.09	- 14.2	- 95.1	23.5	61.3	-100.5	-40
1-F	AK-1	- 20.5	- 28	87.4	7.54	- 97	3.5	32	152	2
1-F	III	- 50.6	- 28	70.91	10.36	-198	- 29.7	199	138	49
1-F	F.G.	17.4	- 28	120.09	- 4.03	-225.1	- 25.9	40	-16.8	-25
1-B	F.G.	- 4.30	- 2	116.72	- 16.56	-148.9	- 9.5	261	- 13	-41
1A-F	#3	26.9	- 27	57.57	15.4	-268	- 15.9	194	157	-39
1A-B	#3	17.5	6	77.37	- 13.09	- 79	14.4	39	- 65	-34
1A-F	Hybrid	- 16.3	- 27	82.85	15.01	- 85	48	35	- 76	-43
2-B	III	- 80	4	84.04	6.02	-114	26.1	104	-155	-52
2-B	F.G.	13.4	4	77.99	- 4.8	- 40	20.7	57	- 20	-46
2-F	III	- 49.6	- 19.5	87.72	10.14	-136	24.9	87.5	-163	44
2-F	F.G.	- 59.9	- 19.5	81.69	7.63	- 59	- 59.7	82	92	-50
3-B	AK-1	12.8	- 1	122.02	- 15.76	-100	15	68	- 61	-42
3-B	#3	60.7	- 1	106.78	- 11.35	-294	8.7	142	-152	-42
3-B	III	34.5	- 1	143.45	- 16.54	-168	15.1	86	- 84	-42
3-B	F.G.	37.4	- 1	117.78	- 8.06	-154	1.2	21	- 90	-20
3-F	AK-1	15.3	- 16	118.79	- 7.77	- 92	10.3	56	- 41	-41
3-F	#3	62	- 16	126.73	- 4.24	-274	- 2.6	88	-171	-38
3-F	III	44.8	- 16	99.1	- 1.09	-117	10	44	- 52	34
3-F	F.G.	39.3	- 16	100.59	1.39	-148	- 15	36	114	-24
4-F	AK-1	- 27.3	- 17	108.63	- 0.2	- 86	1.9	6	126	-34
4-F	#3	109.0	- 17	149.06	- 6.7	-297	- 12	40	163	-39
4-F	Hybrid	- 18.8	- 17	116.97	- .55	-113.7	0.6	15.04	168	-23
5-F	III	46.4	- 30	104.7	- 4.77	-134	- 14	15	2	0
5-F	F.G.	35.4	- 30	88.8	- 11.43	- 83	- 63.9	90	26	-44
6-F	AK-1	- 17.5	- 24	109.21	4.76	- 95.8	- 19.2	48.0	88.4	-38
6-F	#3	18.2	- 24	88.96	1.56	-258	1.6	153	-175	-22
6-F	III	- 14.2	- 24	100.74	- 4.84	-137.5	- 11.2	54	153.4	5
6-F	Hybrid	21.5	- 24	114.02	- 3.1	- 90	5.4	50	- 28	-47
6-B	Hybrid	28.7	18	106.44	- 7.74	- 95.3	33	78	- 47	-39
6-F	F.G.	13.2	- 24	93.48	- 0.96	- 71.1	- 66.2	96	37	-48
7-B	III	13.2	4	98.36	- 2.96	-183	17.5	87	-137	-23
7-B	Hybrid	3.3	4	114.83	- 2.04	- 68	24.6	56	- 42	-32
7-F	AK-1	- 9.92	- 22	90.62	12.06	-101.0	5.8	24.4	149	-21
7-F	#3	- 12.3	- 22	103	16.67	-321	13.4	229	-170	-27
7-F	III	- 20.5	- 22	108.82	15.59	-143	2.1	65	151	- 1
7-F	Hybrid	3.4	- 22	124.03	12.61	- 85	13.9	42	7	-22
7-B	AK-1	- 6.15	4	104.24	- 8.49	-1.8.8	29.5	74.6	- 95	-33

(1) B suffix indicates "Head Back"; F suffix indicates "Head Forward".

(2) PFD Types:

AK-1	- Standard Coast Guard Type II buoyant vest.
#3	- Coast Guard Approved Type I vest (kapok).
F.G.	- Coast Guard Approved Type I vest (fiber glass).
III	- Coast Guard Approved/UL Listed Special Purpose Buoyant Device (Type III).
Hybrid	- Experimental PFD furnished by Coast Guard.

TABLE 13

ERRORS IN CALCULATED EQUILIBRIUM ANGLE ASSOCIATED WITH ERRORS IN ISV AND LV MEASUREMENTS FOR SUBJECT 3

Device	Head Position	Error Caused by One Standard Deviation in: ISV Magnitude	ISV Angle	LV Magnitude
Fibrous Glass	Back	30.2°	1.0°	41.0°
Design 3	Back	2.6°	0.76°	4.8°

TABLE 14

ERRORS IN CALCULATED EQUILIBRIUM ANGLE ASSOCIATED WITH ERRORS IN BV AND LV MEASUREMENTS FOR SUBJECT 3

Device	Head Position	Error Caused by Given Increment: 10% BV Magnitude	5° BV Angle	5° LV Angle
Fibrous Glass	Back	47.4°	0.75°	0.15°
Design 3	Back	3.9°	9.8°	1.9°

TABLE 15

OBSERVED EQUILIBRIUM FLOTATION ANGLES

Subj. No.	Head Pos.	Personal Flotation Device: Fibrous Glass	Design 3	Hybrid	AK-1	Type III
1	B	-41°			-40°	*
	F	-25°			2°	49°
1A	B		-34°	*		
	F		-39°	-43°		
2	B	-46°	*	*	*	-52°
	F	-50°	*	*	*	44°
3	B	-20°	-48°	*	-42°	-42°
	F	-24°	-38°	*	-41°	34°
4	B	*	*	*	*	*
	F	-40°	-39°	-28°	-34°	*
5	B	*	*	*	*	*
	F	-44°	*		*	0°
6	B	*	*	-39°	*	*
	F	-48°	-22°	-47°	-38°	5°
7	B	*	*	-32°	-33°	-23°
	F	*	-27°	-22°	-21°	- 1°

Notes:

B=Head back; F=Head forward.

Negative angle indicates body axis is back from vertical.

Instrument limitation is 64°.

*Angle which exceeds 64° back from vertical position.

TABLE 16

BUOYANCY VECTORS FOR PERSONAL FLOTATION DEVICES

Subj. No.	Head* Pos.	Buoyancy Vector (in.-lb.) AK-1	Design 3	Fibrous Glass	Type III	Hybrid
1	B	95 @ 23.5°		148 @ -9.5°		
1	F	97 @ 3.5°		225 @ -25.9°	198 @ -29.7°	--
1A	B		268 @ 15.9°			
1A	F		79 @ 14.4°			85 @ 48°
2	B	--	--	40 @ 20.7°	114 @ 26.1°	--
2	F	--	--	59 @ -59.7°	136 @ 24.9°	--
3	B	100 @ 15°	294 @ 8.7°	154 @ 1.2°	168 @ 15.1°	--
3	F	92 @ 10.3°	274 @ -2.6°	148 @ -15°	117 @ 10°	--
4	F	86 @ 1.9°	297 @ -12°		--	114 @ 0.6°
5	F	--	--	83 @ -63.9°	134 @ -14°	--
6	B			--		95 @ 33°
6	F	97 @ -19.2°	258 @ 1.6°	--	138 @ -11.2°	90 @ 5.4°
7	B	119 @ 29.5°	--	--	183 @ 17.5°	68 @ 24.6°
7	F	101 @ 5.8°	321 @ 13.4°	--	143 @ 2.1°	85 @ 13.9°

*B = Head Back; F = Head Forward Positions.

Dashed lines indicate data outside range of angle measuring device.

Buoyancy vector magnitude is inherently negative as used in vector diagrams.

TABLE 17

PERCENT OF AVAILABLE BUOYANCY USED BY EACH PFD TO SUPPORT TEST SUBJECT AT OBSERVED EQUILIBRIUM FLOTATION ANGLE

		Personal Flotation Device									
		AK-1		Design 3		Fibrous Glass		Type III		Hybrid	
Subj. No.	Head Pos.*	Deg.	% (18.25)	Deg.	% (35.2)	Deg.	% (30.5)	Deg.	% (15.8)	Deg.	% (16.5)
1	B	-40	**			-41	49	--	--		
1	F	2	41			-25	60	49	79		
1A	B			-34	11					--	--
1A	F			-39	48					-43	84
2	B	--	--	--	--	-46	37	-52	53	--	--
2	F	--	--	--	--	-50	36	44	71	--	--
3	B	-42	62	-42	56	-20	46	-42	84	--	--
3	F	-41	62	-38	51	-24	45	34	54	--	--
4	B	--	--	--	--	--	--	--	--	--	--
4	F	-34	48	-39	48	-64	23	--	--	-28	**
5	B	--	--	--	--	--	--	--	--	--	--
5	F	--	--	--	--	-44	25	0	56	--	--
6	B	--	--	--	--	--	--	--	--	-39	**
6	F	-38	37	-22	49	-48	25	5	63	-47	42
7	B	-33	**	--	--	--	--	-23	72	-32	18
7	F	-21	81	-27	52	--	--	-1	59	-22	28

Dashed lines indicate off scale readings (equilibrium angle greater than 60 degrees).

Negative angles indicate body axis back of vertical.

( ) Buoyancy of totally submerged PFD in pounds.

*B= Head Back; F=Head Forward.
**Data not determined.

FIG I

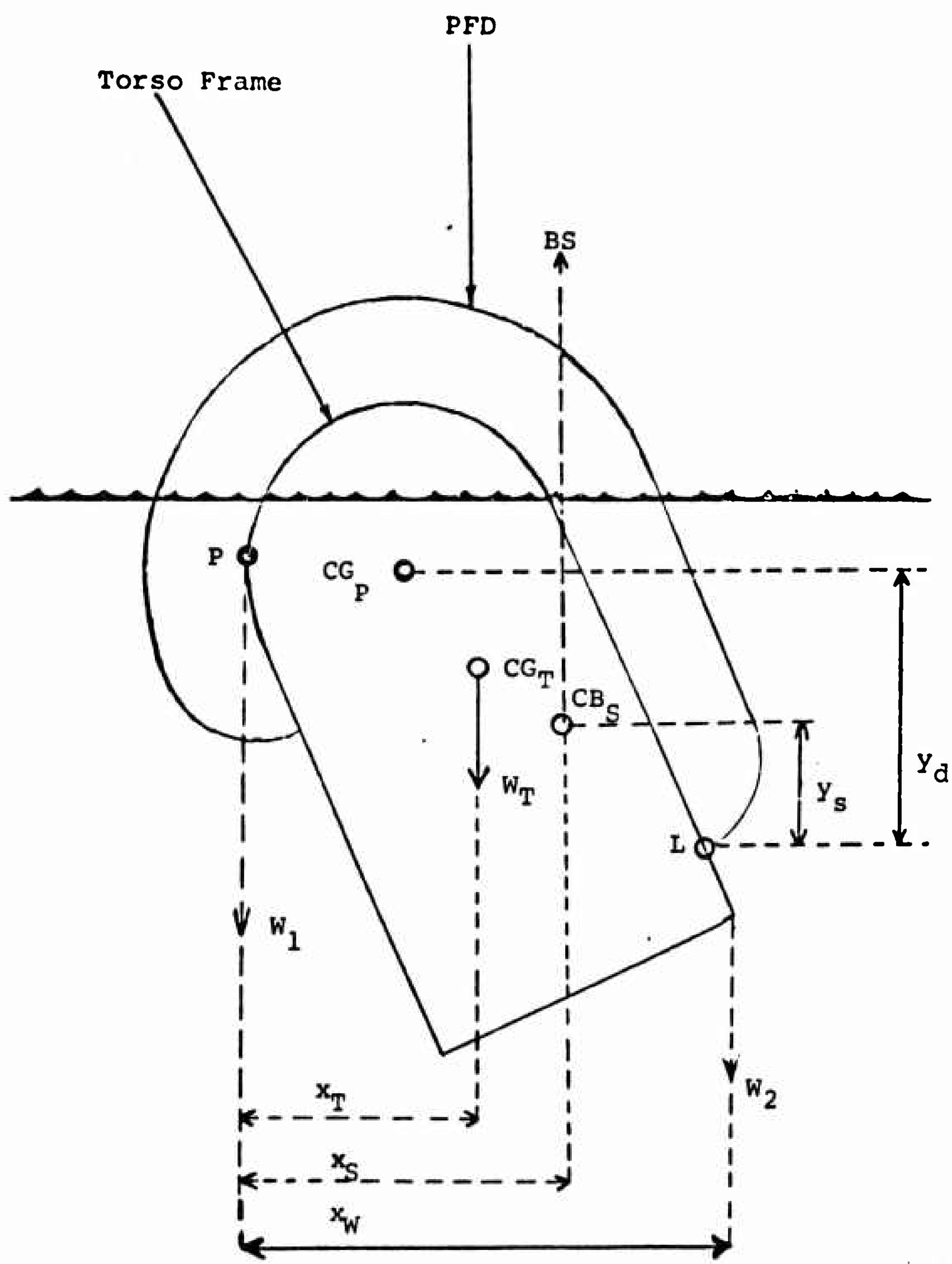


FIGURE 3

SCHEMATIC REPRESENTATION OF THE TECHNIQUE FOR DETERMINING BUOYANCY AND CENTER OF BUOYANCY OF PARTIALLY EMERGED PFD'S

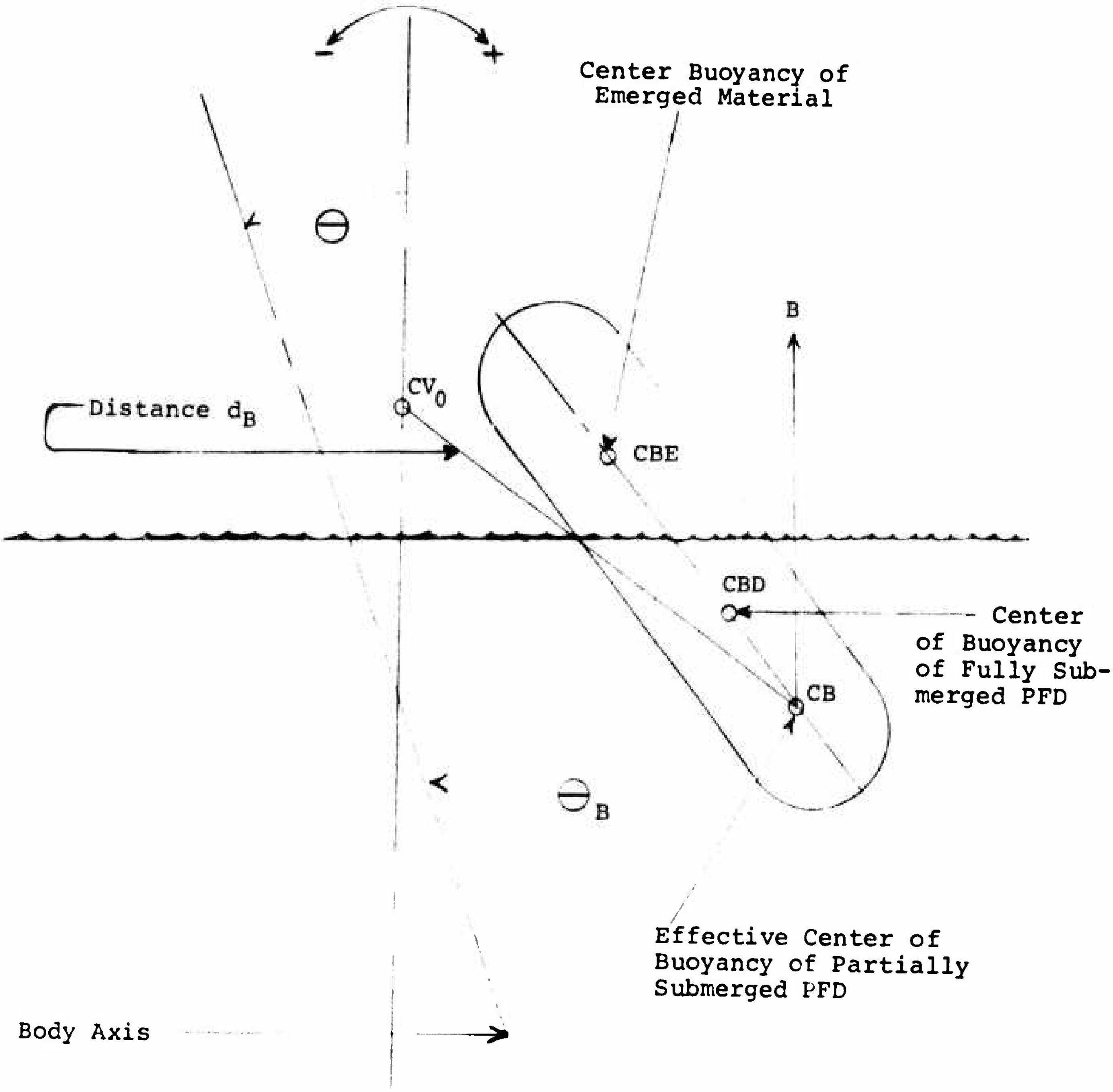


FIGURE 4

SCHEMATIC DIAGRAM OF PARTIALLY EMERGED PFD

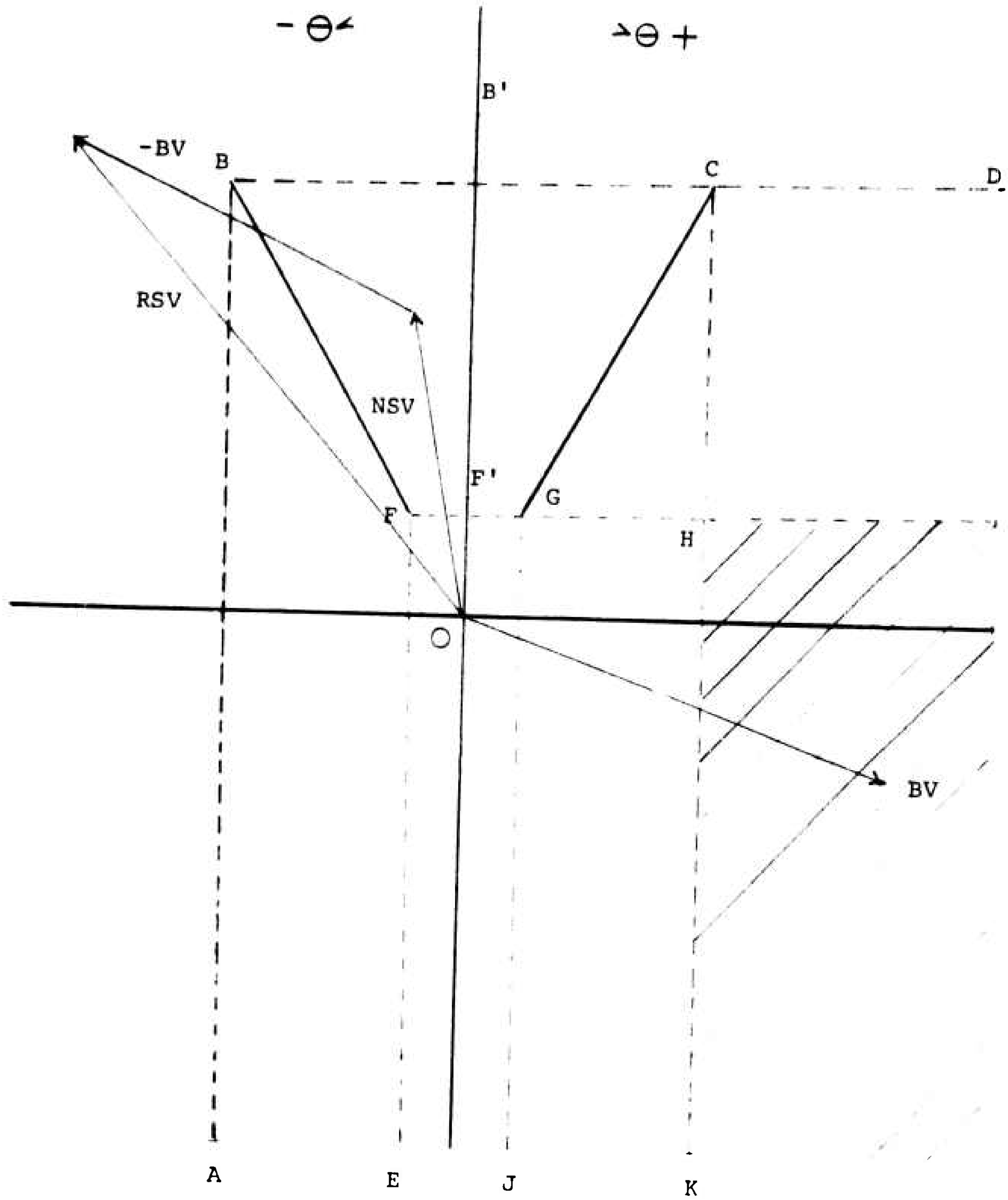


FIGURE 5

SCHEMATIC REPRESENTATION OF LIMITING VALUES OF THE BUOYANCY VECTOR

REFERENCES

1. Anon., "Final Report on Buoyancy and Stability Characteristics of the Human Body in Fresh Water", Arthur D. Little, Inc., Project 725106, Coast Guard Contract DOT - CG - 90, 511 - A (April 13, 1972).

2. Natrella, M. G., "Experimental Statistics", National Bureau of Standards Handbook 91 (August 1, 1963), pp. 3-26 to 3-28.

3. Ibid., pp. 4-8 to 4-9.

4. Ibid., pp. 2-7.

5. Ibid., pp. 4-9 to 4-10.

APPENDIX A

FLOTATION THEORY AND DEFINITIONS
(Extracted From Reference 1)

IV. STABILITY

A. THEORETICAL ANALYSIS

This analysis will deal with fore-and-aft rotations only and is therefore directed to those classes of personnel flotation device which are transversely symmetric, that is, there is no unequal lateral distribution across the chest or back.

1. Head Back

Consider an individual in flotation equilibrium, though not necessarily in rotational equilibrium, under the condition that:

- his head is tilted back
- his lungs are filled to functional residual capacity
- he is equipped with a personnel flotation device providing an amount of buoyancy $B(\theta)$ at a distance d_B from the center of volume floated above water, and at an angle θ_B to the body centerline
- he is inclined to the vertical by an angle θ

Under these conditions, the subject will have five vertical forces acting on him. These forces are diagrammed in Figure IV-1 and are specifically:

$\rho_w V_o$ - The loss of buoyancy due to bringing the volume V_o above water, acting vertically downward through $CV_o(\theta)$ - the center of floated volume.

W - The subject's weight in air acting vertically downward through $CG(\theta)$ - the subject's center of mass.

$\rho_w V_T = W$ - The buoyancy of the total body acting vertically upward through $CV_T(\theta)$. Equal to the subject's weight when the lungs are inflated to provide zero net buoyancy.

$\rho_w \Delta V_L$ - The change in buoyancy provided by the lungs, in changing inflation from that yielding zero net buoyancy to functional residual capacity, acting vertically upward through $C\Delta V_L$ - the center of change of buoyancy of the lungs.

$B(\theta)$ - The buoyancy provided by the personnel flotation device acting vertically upward through $CB(\theta)$ - the effective center of buoyancy of the device.

In each of the definitions above, where a buoyancy or a center of buoyancy may change its value or location with changing inclination angle, the dependence on θ is indicated.

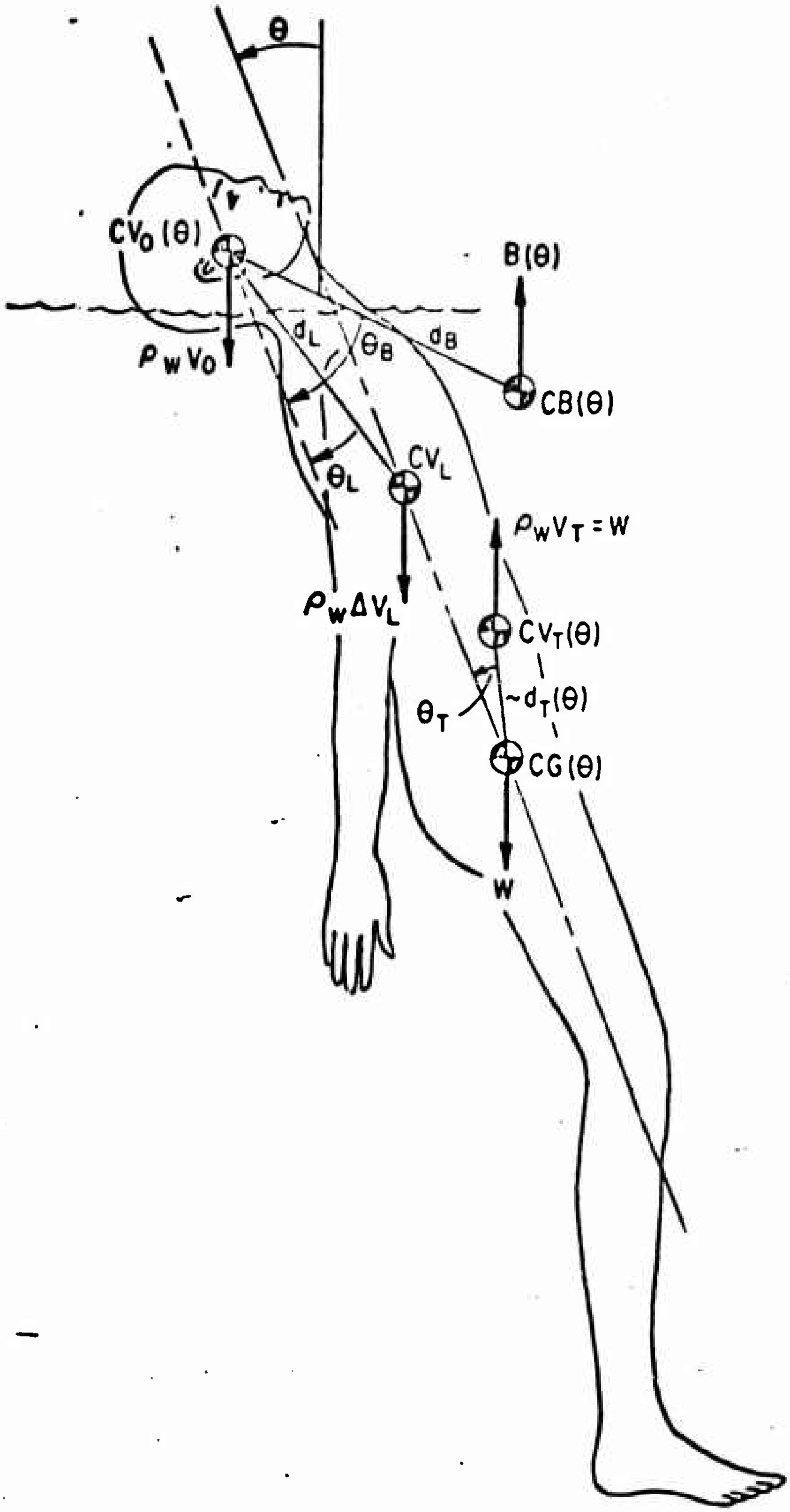


Figure IV-1 Forces Acting on a Subject Equipped with a Personnel Flotation Device - Head Back

Referring to Figure IV-1 we can now take moments about the center of floated volume $CV_o(\theta)$ and obtain an expression for the turning moments acting on the subject when he is inclined at an angle θ to the vertical. This expression is of the form

$$M(\theta) = Wd_T(\theta)\sin(\theta - \theta_T) + \rho_w \Delta V_L d_L \sin(\theta - \theta_L) - B(\theta)d_B(\theta)\sin(\theta - \theta_B) \qquad \text{(IV-1)}$$

where

$M(\theta)$	is the total turning moment acting when the subject is inclined at the angle θ to the vertical.
$Wd_T(\theta)\sin(\theta - \theta_T)$	is the individual's <u>zero buoyancy moment</u> - defined as that moment that would act on the subject without a personnel flotation device when he is at the inclination angle θ under the condition that his lungs are inflated to yield zero net buoyancy.
$\rho_w \Delta V_L d_L \sin(\theta - \theta_L)$	is the individual's <u>lung moment</u> defined as the moment that acts on the subject at the inclination angle θ, due to changing lung volume from that yielding zero net buoyancy to functional residual capacity.
$- B(\theta)d_B(\theta)\sin(\theta - \theta_B)$	is the <u>device moment</u> - defined as the moment provided by the personnel flotation device when the subject is at the inclination angle θ.

The distances $d_T(\theta)$, d_L, $d_B(\theta)$ as well as the angles θ_T, θ_L and θ_B are illustrated in Figure IV-1.

2. Diagrammatic Representation

It is possible to interpret each of the three moment terms on the right-hand side of equation IV-1 as the horizontal components of a vector. This is a convenient way of treating the dependence of moments on body orientation and added buoyancy location. This interpretation is possible, since the five forces acting are parallel. The three vectors are defined as:

1. The intrinsic stiffness vector
   magnitude $Wd_T(\theta)$ at an angle $(\theta - \theta_T)$ From the Vertical

2. The lung vector
   magnitude $\rho_w \Delta V_L d_L$ at an angle $(\theta - \theta_L)$ "

3. The buoyancy vector
   magnitude $B(\theta)d_B(\theta)$ at an angle $(\theta - \theta_B)$ "

It follows that the total moment $M(\theta)$ can be interpreted as the sum of the three horizontal components of these vectors or as the horizontal component of the sum of the three individual vectors. The sum vector will be defined as the resultant stiffness vector.

a. Vertical Subject

The vector diagram for the special case of a vertical subject, that is, $\theta = 0$, with head back, is shown in Figure IV-2. In this diagram, the sum of the three vectors is the resultant stiffness vector, and its horizontal component is $M(0)$, the turning moment acting on the vertical subject. Note that the lung vector magnitude can be either positive or

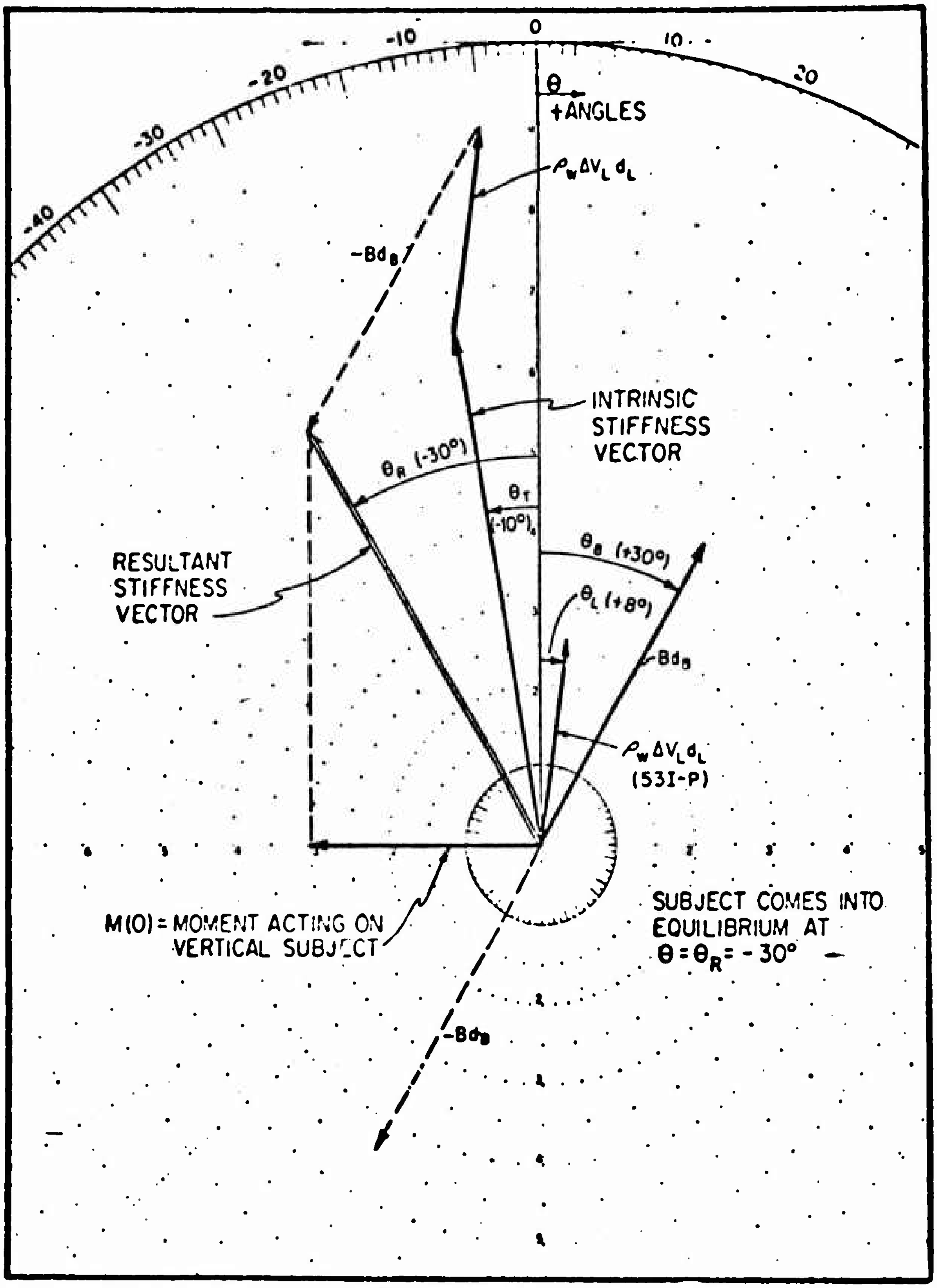


Figure IV-2 Diagrammatic Representation of Turning Moments - Head Back

negative, since the quantity $\rho_w \Delta V_L$ can have either sign, and that the buoyancy vector magnitude is intrinsically negative. A negative magnitude simply causes a 180^0 rotation of the vector. If M(0) is positive, that is, pointing to the left, as in the case illustrated, the subject will be rotated counterclockwise. Conversely, if M(0) is negative, that is, pointing to the right, the subject will be rotated clockwise. In either case, the rotation will continue until an equilibrium angle is attained such that $M(\theta) = 0$, that is, until his resultant stiffness vector is vertical.

b. Arbitrary Orientation

In those cases where the inclination is angle, θ, is other than zero, each of the three vectors that determine the resultant stiffness vector and the total moment acting would be rotated through the angle θ. If the magnitudes $Wd_T(\theta)$, $\rho_w \Delta V_L d_L$ and $B(\theta)d_B(\theta)$ as well as the relative phases θ_T, θ_L and θ_B of these three vectors are assumed to be independent of θ, the net result of a change in inclination angle, θ, would be a rotation of Figure IV-2 through an angle θ. The magnitude of the resultant stiffness vector would be unchanged; however, its horizontal component, that is, the total moment acting on the subject, would vary with θ. The subject will be rotated in the counterclockwise direction if $M(\theta)$ is positive, and clockwise if $M(\theta)$ is negative. He will be at his rotational equilibrium position when $M(\theta)$ is zero, corresponding to the case where the resultant stiffness vector is vertical. This occurs in the case illustrated in Figure IV-2 when the subject is inclined backwards at an angle of 30^o.

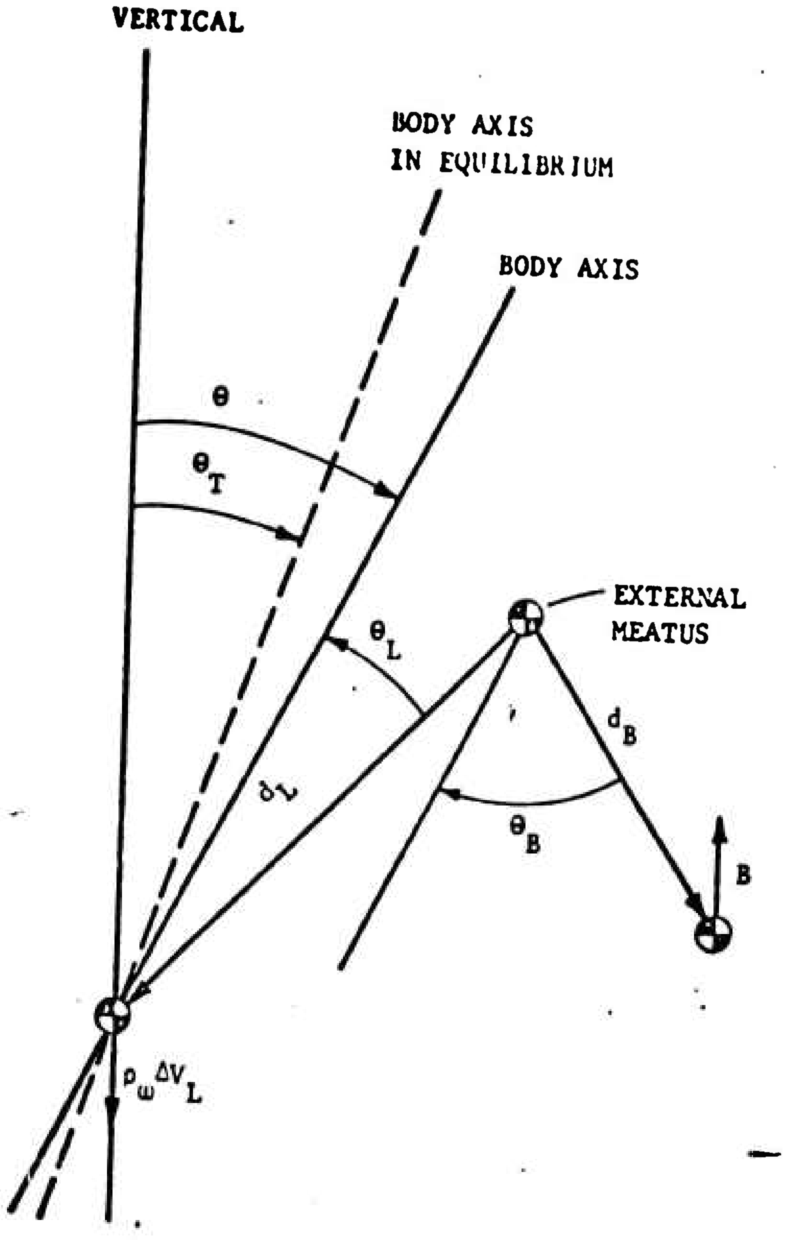


FIGURE V-1 VECTOR SIGN CONVENTION

All angles are taken as positive clockwise. The intrinsic and resultant stability vector angles are measured from the vertical to the body axis in rotational equilibrium. The lung and buoyancy vector angles are measured from the vector to the body axis. Since the lung vector magnitude is computed as $\rho_\omega \Delta V_L d_L$ it can be either positive or negative, depending on the individual.

DEFINITIONS

B - Buoyant force (lbs.) - The additional force required to support a given volume (V_o) of an individual above water. In thecretical calculations using the theory, B equals the net upward force exerted by a PFD on an individual.

BV - Buoyancy vector - A vector whose horizontal component is the turning moment produced by the flotation device. Its magnitude is the product of the buoyancy of the PFD and the distance from the center of buoyancy (CB) of the PFD to the center of the floated volume (CV_o) of the subject.

CB - Center of buoyancy.

CG - Center of gravity.

CV_L - Center of buoyancy of the lung volume.

CV_o - Center of volume of the above water portion of the subject's body.

FRC - Functional Residual Capacity - The volume of air contained in the lungs at the bottom of the normal breathing cycle. Used as the reference state of lung inflation for experimental and analytical purposes.

ISV - Intrinsic Stiffness Vector - A vector whose magnitude is the product of the subject's weight (W) and the distance between the subject's CG and CB when fully submerged with the lungs inflated to provide zero net buoyancy. The vector angle is measured between this line and the vertical. The horizontal component of this vector is the moment that would act on the subject without a PFD when he is at any given angle under the condition zero net buoyancy.

LV - Lung vector - A vector whose horizontal component is the turning moment produced by a change in lung volume ΔV_L from the condition of zero net buoyancy to the condition under consideration. The magnitude of LV is the product of the loss in buoyancy due to ΔV_L and the distance between the centers of the lung volume and floated volume. The vector angle is measured between the line joining those centers of volume and the vertical.

NSV - Net Stiffness Vector - The vector sum of the ISV and LV.

PFD - Personal Flotation Device.

ρ_w - Density of water. Assumed to be a constant 62.4 lbs./cu.ft.

RSV - Resultant Stiffness Vector - The sum of the NSV and BV.

SSN - Suprasternal notch.

V_T - Subject's total volume including lungs when lungs are inflated so that $\rho_w V_T = W$.

W - Weight of an individual in air.

θ - Equilibium Flotation Angle. Same as angle of Resultant Stiffness Vector. It is the angle measured in the right profile view of the body from the vertical to the body axis when the body is in rotational equilibrium. Angles measured clockwise from vertical are positive. Also referred to as angle of repose and as equilibrium angle.

A P P E N D I X   B

EXAMPLE OF DETERMINING THEORETICAL EQUILIBRIUM FLOTATION ANGLE (I.E. THE ANGLE OF THE RSV) USING SUBJECT NO. 1A, (BEFORE BREAKFAST TEST SESSION), HEAD FORWARD, IN DESIGN NO. 3 PFD

I. Use computer program CB2 for determining buoyancy and x, y coordinates from lowermost, outermost point on PFD as worn on torso.

A. Data input into computer (inches and degree):

WDD=WT OF DRY SAMPLE DEVICE	2.95
HVT=HOR DIST FR FRONT OF VEST TO FRONT OF TORSO	4.75
VVT=VERT DIST FR BOTTOM OF TORSO TO BOTTOM OF VEST	0
HS=HOR DIST FR FRONT OF VEST TO SYSTEM CG	9.6
VSI=VERT DIST FR BOTTOM OF TORSO TO CG OF SYSTEM	10.4
BVT=BUOYANCY OF TOTALLY SUBMERGED DEVICE	35.18
WT11=WT OF TORSO FRAME PARTIALLY EMERGED	4.7
ALPHA=ANGLE FR HORIZ OF TORSO W/PARTIALLY EMERGED PFD	51
W1=WT AT TORSO FRONT TO GIVE ALPHA	8.28
W2=WT AT TORSO ARM TO GIVE ALPHA	4.03
VWT=EST. VERT DIST FR TORSO BOT TO CG OF ABOVE-WATER TORSO	15
HWT=EST HOR DIST FR TORSO FRONT TO CG OF ABOVE-WATER TORSO	3
L11=HOR DIST FR TORSO FRONT TO LOWEST POINT OF BACK FLOTATION MAT'L--IF NO MAT'L ON BACK, SET L11&VVTT=0	11
VVTT=VERT DIST FR TORSO BOT TO LOWEST POINT OF BACK	13

B. Computer results:

EFFECTIVE BUOYANCY OF PFD (LBS) = 17.01 (48.4% OF FULL VALUE)
HOR DIST FRO FRONT OF PFD TO CB = 8.57
VERT DIST FRO BOTTOM OF PFD TO CB = 5.83
HOR DIST, IN., FR FRONT OF DEVICE TO DEVICE CG = 9.51
VERT DIST FR BOTTOM OF DEVICE TO DEVICE CG = 9.08

II. Use computer program ULBV to compute the buoyancy vector. This program is needed to relate the coordinate system in Step I above, to the subject's body; i.e., the distance from the CB to the center of the floated volume,assumed to be at the external meatus.

A. The following data (inches, degree and pounds) is needed for the program.

E=CHEST THICKNESS =	12
F=VERT DISTANCE FROM SUPRASTERNAL NOTCH (SSN) to CENTER OF LUNG VOLUME =	7
DL2=DISTANCE FROM CENTER OF LUNG VOLUME TO EXTERNAL MEATUS, HEAD FORWARD =	14
TL2=ANGLE FROM DL2 TO BODY AXIS (CLOCKWISE BEING POSITIVE) =	-27°
T1=PFD THICKNESS	4.63
Y22=VERTICAL DISTANCE FROM SSN TO PFD BOTTOM =	15.5
B2=EFFECTIVE BUOYANCY FROM STEP I =	17.01
Y1=VERTICAL COORDINATE OF PFD (FROM STEP I)=	5.83
X1=HORIZONTAL COORDINATE OF PFD =	8.57

B. <u>Computer results</u>:

BV MAGNITUDE = 268 in-lbs.
BV ANGLE = -15.8 deg.

It is to be noted that the BV magnitude is intrinsically negative in the A.D. Little theory.

III. Determine the Intrinsic Stiffness Vector, ISV. Use computer program ULSTAB to compute ISV. This program was written by A.D. Little, Inc. and adopted to the UL Time-Share Computer. The subject was fitted with the test harness (Fig. VII-2) to which was attached an angle measuring device and a weight-float combination that exerted a couple. The value of the torque applied to the subject depended on the separation between the weight and float and the angle between the vertical and the line joining the attachment points.

A. The following data was input to the computer.

(1) The weight in water, pounds, at the weight used to oppose a buoyant ball having a buoyancy equal in magnitude to the weight. This was 7.5 lbs.

(2) The value of the separation, in inches, between the buoyant ball and weight, and the corresponding angle from the vertical. A couple tending to rotate the subject backwards was input as a positive value. Angles forward from the vertical were also considered positive.

(3) The angle from the vertical at zero torque and with the subject standing on a submerged platform. This was a base point to be subtracted from each observed angle.

(4) The following is the actual computer input with some brief explanatory information:

NAME -BB2

2,8 ←—— Subject No. and Day
7,5,1 ←— Weight under water and base angle
0,1 ←—— Distance between weight float and observed angle with vertical
0,0
-2,-4,5
-4,-9
-6,-13
-8,-17
-10,-20,5
-12,-19.5
+2,+11
+4,+13
999.0 ←— End of data signal

B. Computer Results:

ISV Magniture = 81.69 in-lbs.
ISV Angle = 7.63 deg.

IV. Determine the Lung Vector, LV. The difference between zero net buoyancy and the condition where the subject held his breath is found as follows:

A. ΔLV for the FRC condition, i.e. from zero net buoyancy to FRC, is determined by means of the respirometer on the day of the test. This value was 1.46 liters.

B. ΔLV from the FRC condition to the point where the subject held his breath was then determined. In this case the value was 0.59 liters.

C. The difference between the results of A and B, above, was 1.46-0.59=.87 liters.

D. The lung vector was then determined by combining:

(1) The measured distance between the lung center, CVL, and external meatus, CVO, which was 14 inches.

(2) The angle between this line and the body axis, which was -27 degree, i.e., forward of the body axis.

(3) The lung vector magnitude was then 2.2046 lbs/liter X 0.87 liters X 14 in.=26.9 in.-lb. at -27 deg.

V. The Resultant Stiffness Vector, RSV, the angle of which was the predicted equilibrium flotation angle, was found by using the computer program ULADV as follows:

A. The following data was input to the program ULADV:

(1) ISV data from Table III-3
NS=Subject No. 1
A=ISV magnitude=57.57
TH=ISV angle=15.4 deg

(2) The lung vector for the subject as determined in Step (3) above:
NSL=Subject No. 1
AL=26.9
THL=-27

(3) The computer output was the net stiffness vector, NSV, having a magnitude equal to 79.5 in.lb. at an angle of 2.2 deg.

(4) The NSV is an intermediate step leading to the RSV. ULADV was therefore used to add the NSV to the BV as follows:

(a) Input the NSV into the program as
NS=Subject No. 1
A=NSV magnitude=79.5
TH=2.2

(b) Input the BV from Step (1) above, as
NS=Subject No. 1
AL=268
THL=-15.9

(c) The computer output is the RSV with a magnitude = 344.5 at an angle at -11.6 deg.

APPENDIX C

Computer Programs and Subroutines

Listings of the following computer programs and subroutines used in the investigation are given in this Appendix:

1. CB of Partially Emerged PFD
2. Subroutine CGPFD
3. Subroutine CGMOVE
4. ULBV
5. ULSTAB
6. ULADV

Documentation is also given for the first three items above. The programs ULBV and ULSTAB are identical to those used in the investigation reported in Reference 1 and documentation is not repeated here. ULBV determines the magnitude and phase angle of the buoyancy vector from measurements of individual subjects and PFD's, ULSTAB computes the equilibrium flotation angle for a subject wearing a PFD, given the magnitudes and phase angles of the intrinsic stiffness vector, lung vector, and buoyancy vector.

The last program, ULADV, merely adds vectors in polar form.

PROGRAM - CB OF PARTIALLY EMERGED PFD

This program determines the net buoyant force and locates the center of buoyancy of the PFD in the partially emerged position. Figures C-3 and C-4 show the geometric relationships and symbols used. The required inputs are:

HVT - Horizontal distance from the front of the PFD to the front of the torso.

VVT - Vertical distance from the bottom of the torso to the bottom of the PFD.

HC - Horizontal distance from the front of the torso to the point W_2 is applied.

VC - Vertical distance from the bottom of the torso to the point W_2 is applied.

BVT - Net buoyancy of the totally submerged PFD.

WT11 - Net weight of torso frame partially emerged.

α - Angle between horizontal and torso axis.

W1 - Weight applied at torso front.

W2 - Weight applied at torso arm.

L11 - Horizontal distance from torso front to lowest point on back flotation material.

VVTT - Vertical distance from torso bottom to lowest point on back flotation material.

(If no flotation material is on back of PFD, L11 = VVTT = 0)

By force balance, the net buoyancy is:

$$BS = WT11 + W1 + W2$$

The following series of calculation determines the horizontal distance of the CB from the front of the PFD (HCBS) and the vertical distance of the CB from the bottom of the device, (VD1), Subroutines CGMOVE and CGPFD are called during the computation.

First, the location of the vertical line throuch the CB is found by computing moments about the point 0 in Fig. C-3, the point at which force W_2 is applied.

$$H1 = (HC) \sin\alpha$$

$$H2 = (VC) \cos\alpha$$

$$L = (HT11) \sin\alpha + (VT11) \cos\alpha$$

where HT11 and VT11 are obtained from Subroutine CGMOVE.

$$C = H1 + H2 - L$$

The horizontal distance AB from point 0 to the vertical line through CB is now obtained by setting the sum of the moments about point 0 equal to zero.

$$AB = (WT11)(C) + (W1)(H1 + H2)/BS$$

$$BC = H1 + \text{[illegible]} - AB$$

$$L1 = (BC)/\sin\alpha$$

The following steps determine the upper and lower limits, within which CB must fall. The upper limit is simply the level of the CG of the dry device, located by the coordinate CB1 in Fig. C-4. If $\alpha = 0$, CB1 is given by

$$CB1 = HVT - HD$$

where HD is obtained from Subroutine CGPFD. In this case, VBOT = 0.

When α is not zero, the following system of calculations is used to find CB1.

$$VS2 = VD + VVT$$

where VD is obtained from Subroutine CGPFD.

$$HT_4 = HD - HVT$$

$$HSH = (VS2)/\sin\alpha$$

$$A = (L1)\tan\alpha - VS2$$

$$B = (HT4)/\tan\alpha$$

Depending on the value of α, the distance A may be negative, zero, or positive. Fig. C-4 depicts the situation for positive A, for which

$$V1 = (A + B) \sin\alpha$$

This also applies when A = 0. For negative values of A, the situation depicted in Fig. C-5 exists. In that case,

$$C = \left| A/\tan\alpha \right|$$

$$V1 = (HT4 + C) \cos\alpha$$

Calculations are now made to locate the lowest portion of the PFD, which is the lower limit of CB. For material on the front of the PFD

$$V2 = (L1) \cos\alpha$$

$$VBOT = V2 + (VVT) \sin\alpha$$

For material on the back of the PFD,

$$LB = L11 - L1$$

$$VVV = (LB)/\tan\alpha$$

$$VBOT1 = (VVTT - VVV) \sin\alpha$$

If VBOT1 $<$ VBOT, it is substituted for VBOT in the remainder of the computations. Otherwise VBOT for the front material is used. When there is no back material on the PFD, the computation of VBOT1 is omitted. This command is given in the program by setting L11 equal to zero in the input data.

The value of CB1 is now calculated. When A is negative,

$$CB1 = HSH - V1 - VBOT$$

and when A is zero or positive,

$$CB1 = (L1)/\cos \; - V1 - VBOT$$

The quantities VBOT and CB1 locate the lower and upper limits of CB, and it remains to determine CBSHIF by the approximate procedure described in Section II and Fig. 2. Figure C-2A depicts the proportioning relationship graphically, and the appropriate algebraic expression is

$$CBSHIF = \frac{(CB1)\ (BS)}{(WDD + BVT)} + CB1 + VBOT - \frac{(CB1)\ (BVT)}{(WDD + BVT)}$$

It follows then that when $\alpha \neq 0$.

$$HCBS = HVT + L1 - (CBSHIF) \cos \alpha$$

and

$$VD1 = (CBSHIF) \sin \alpha - VVT$$

For the case $\alpha = 0$ however,

$$HCBS = HVT - CBSHIF$$

$$VD1 = VD$$

Thus the magnitude of the net buoyant force BS and coordinates (HCBS, VD1) of the center of buoyancy of the partially emerged device have been determined.

Subroutine CGPFD

This subroutine calculates the location of the center of gravity of the PFD from the coordinates of the CG's of the torso alone and of the torso-PFD system. Fig. C-6 shows the geometric relationships and symbols used. Required inputs are:

HT1 - Horizontal distance from the front of the torso frame to its CG.

HVT - Horizontal distance from the front of the torso to the front of the PFD.

HS - Horizontal distance from the front of the PFD to the CG of the torso-PFD system.

VT - Vertical distance from the bottom of the torso to its CG.

VVT - Vertical distance from the bottom of the torso front to the bottom of the PFD front.

VS1 - Vertical distance from the bottom of the torso to the CG of the torso-PFD system.

The following series of calculation is for the determination of the horizontal and vertical coordinates of the CG of the PFD with respect to its front and bottom surfaces.

$$HT = HT1 + HVT$$

$$VS = VS1 - VVT$$

$$V = VS1 - VT$$

$$H = HT - HS$$

$$H1 = (V^2 + H^2)^{\frac{1}{2}}$$

$$\alpha = \arctan (V/H)$$

$$H2 = WT/(WD) \ (H1)$$

If V is negative,

$$VD = VS - (H2) \sin\alpha$$

If V is zero or positive,

$$VD = VS + (H2) \sin\alpha$$

If H is negative,

$$HD = HS + (H2) \cos \alpha$$

If H is zero or positive,

$$HD = HS - (H2) \cos \alpha$$

Thus, HD and VD, the desired coordinates, are determined.

Subroutine - CGMOVE

This subroutine computes the coordinates (HT11 and VT11) of the effective center of gravity and the net weight (WTUNDR + ABVWTR) of the partially submerged torso. Figure C-1 shows the geometric relationships and symbols used. Required inputs are:

HT1	-	Horizontal coordinate of the true CG of the torso
VT	-	Vertical coordinate of the true CG of the torso
HWT	-	Estimated distance from torso front to the CG of the above-water torso
VWT	-	Estimated distance from the torso bottom to the CG of the above-water torso
WT	-	Weight of torso frame in air
WT11	-	Net weight of torso frame partially submerged
WTS	-	Net weight of torso frame fully submerged
VVT	-	Distance from bottom of torso to bottom of PFD

When there is no above water torso, HWT is set equal to zero and the original CG_T is retained by setting

$$VT = VT11$$

$$HT1 = HT11$$

When there is above water torso, HWT and VWT are estimated and then

$$HT = HT1 + HVT$$

$$VTT = VT - VVT$$

$$V = VWT - VT$$

$$H = HT1 - HWT$$

$$ANG = \left| \arctan V/H \right|$$

$$HTOT = (V^2 + H^2)^{\frac{1}{2}}$$

The weight of the above water torso is calculated as shown in Fig. C-2B, that is:

$$ABVWTR = WT(WT11 - WTS)/(WT - WTS)$$

Similarly, the net weight of the underwater torso is calculated as shown in Fig. C-2C, that is:

$$WTUNDR = WTS\ (WT - ABVWTR)/WT$$

Since the sum of the moments about the new CG is zero,

$$(\mathrm{H1})\ (\mathrm{WTUNDR}) = (\mathrm{HTOT} - \mathrm{H1})\ (\mathrm{ABVWTR}) \text{ or}$$

$$(\mathrm{H1}) = (\mathrm{HTOT})\ (\mathrm{ABVWTR})/(\mathrm{WTUNDER} + \mathrm{ABVWTR})$$

If H is negative,

$$\mathrm{HT11} = \mathrm{HT1} + (\mathrm{H1}) \cos (\mathrm{ANG})$$

If H is positive or zero,

$$\mathrm{HT11} = \mathrm{HT1} - (\mathrm{H1}) \cos (\mathrm{ANG})$$

If V is negative,

$$\mathrm{VT11} = \mathrm{VT} - (\mathrm{H1}) \sin (\mathrm{ANG})$$

If V is positive or zero,

$$\mathrm{VT11} = \mathrm{VT} + (\mathrm{H1}) \sin (\mathrm{ANG})$$

Thus, the CG and downward force associated with the partially submerged torso have been determined.

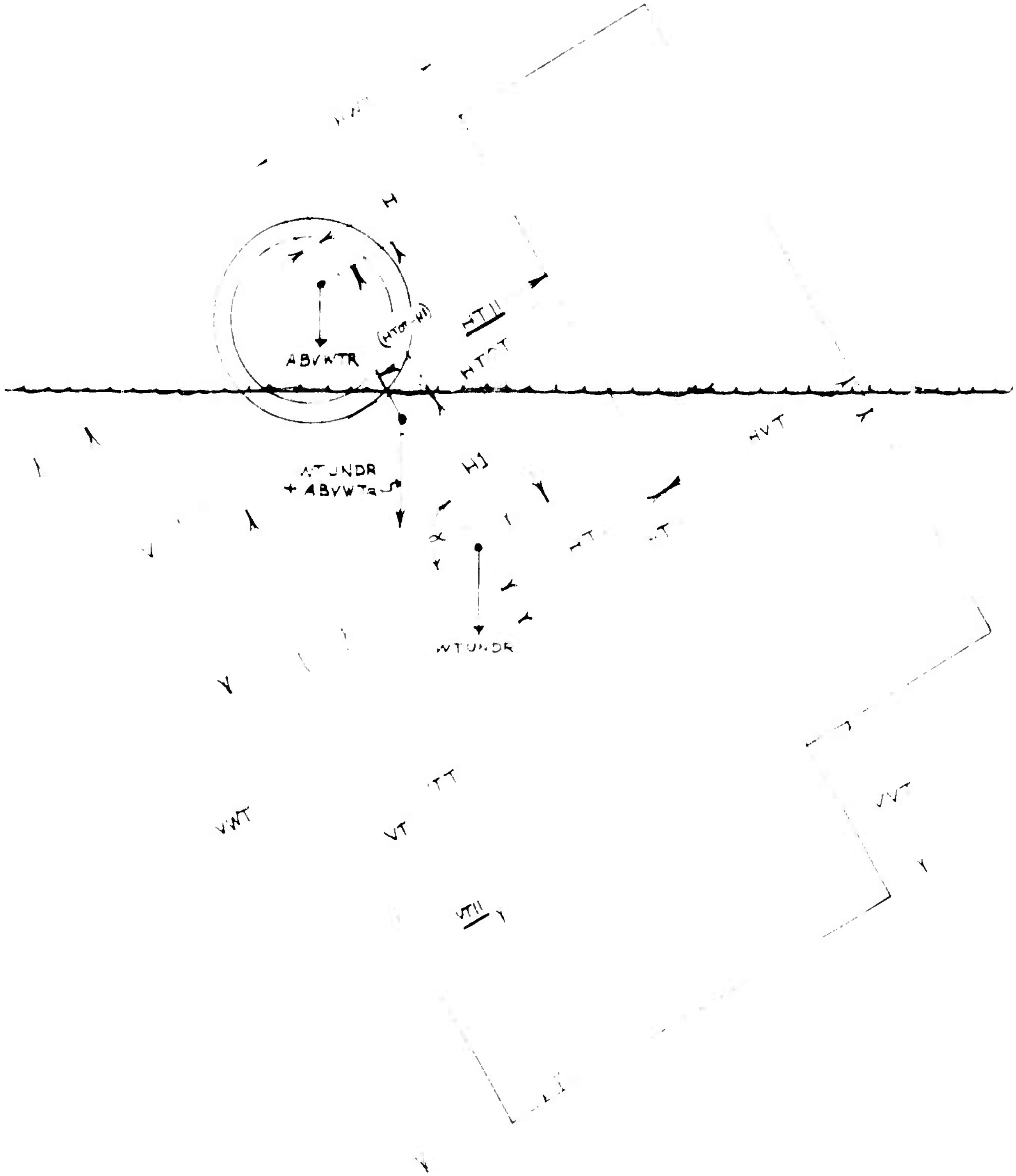
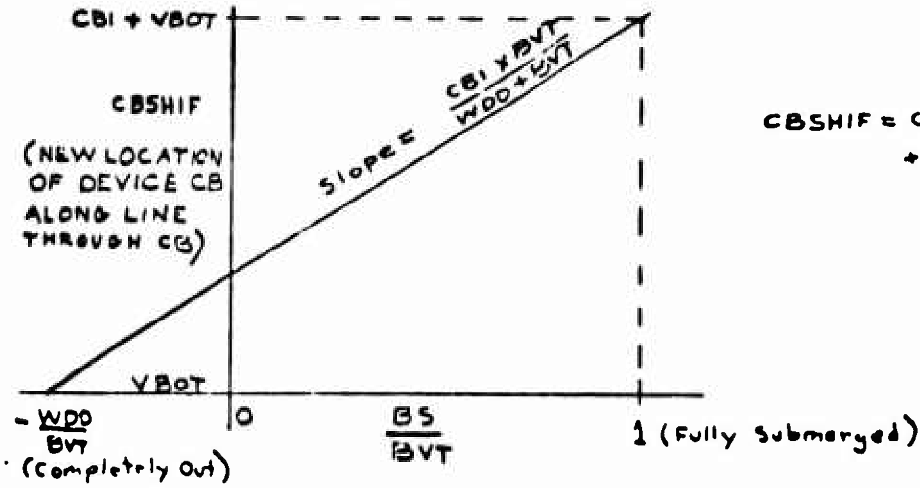


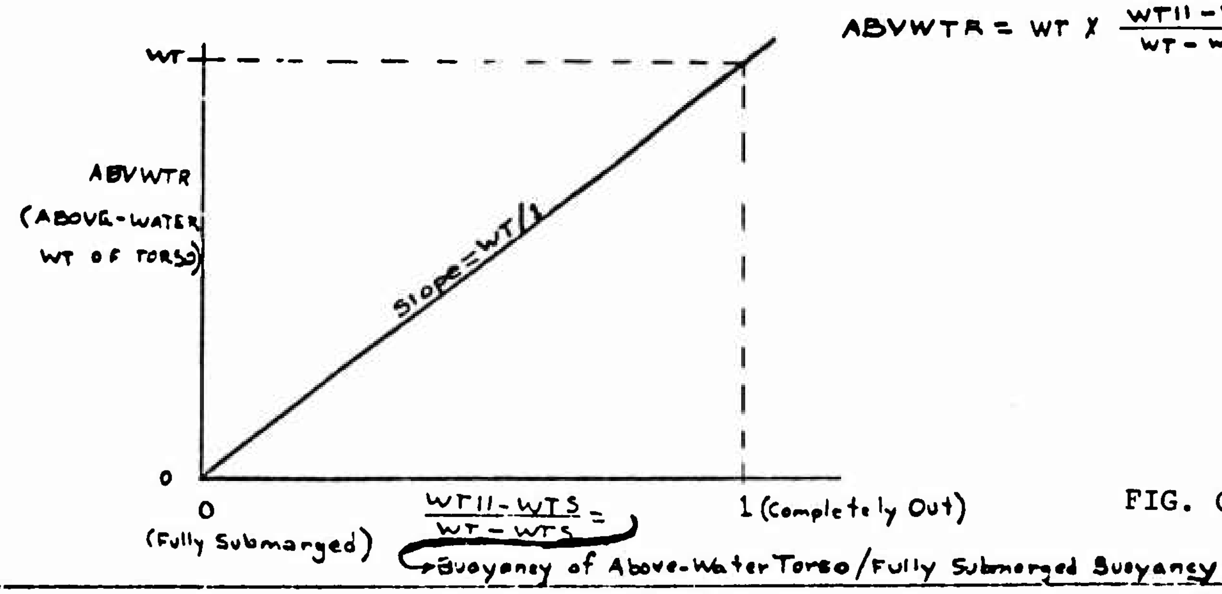
FIGURE C-1

CG AS AFFECTED BY ABOVE-WATER TORSO



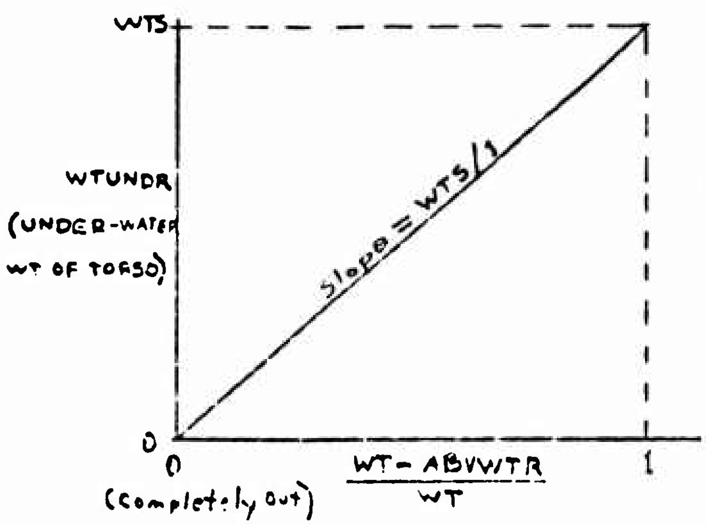
$$CBSHIF = CBI/(WDO+BVT) \times BS + CBI + VBOT - \frac{CBI \times BVT}{BVT + WDO}$$

FIG. C-2A



$$ABVWTR = WT \times \frac{WT11 - WTS}{WT - WTS}$$

FIG. C-2B



$$WTUNDR = WTS \times \frac{WT - ABVWTR}{WT}$$

FIG. C-2C
ILLUSTRATION OF
CALCULATIONS OF CB AND CG

ANALYSIS OF PRELIMINARY EQUATIONS

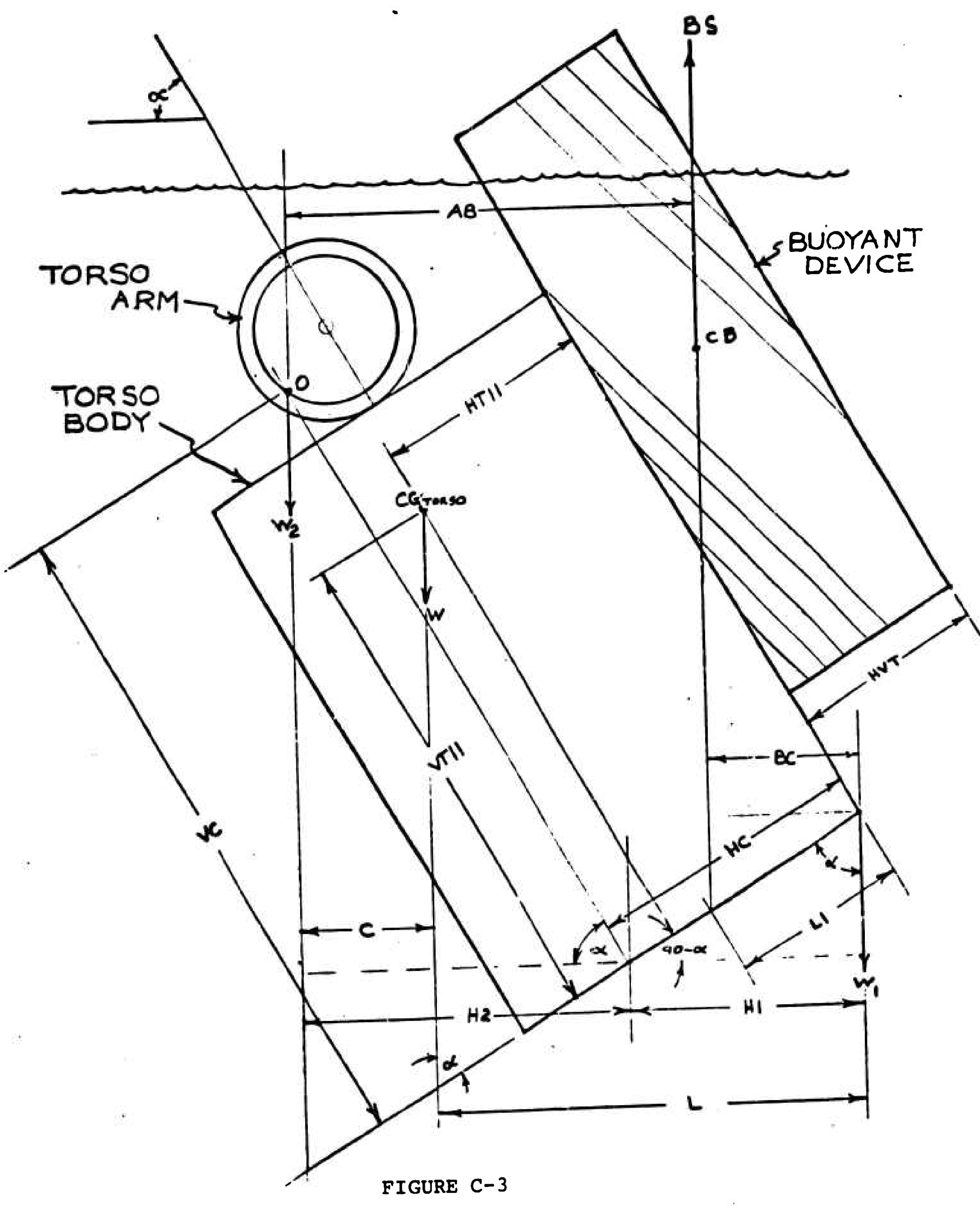


FIGURE C-3

SCHEMATIC OF PFD - TORSO FRAME SYSTEM

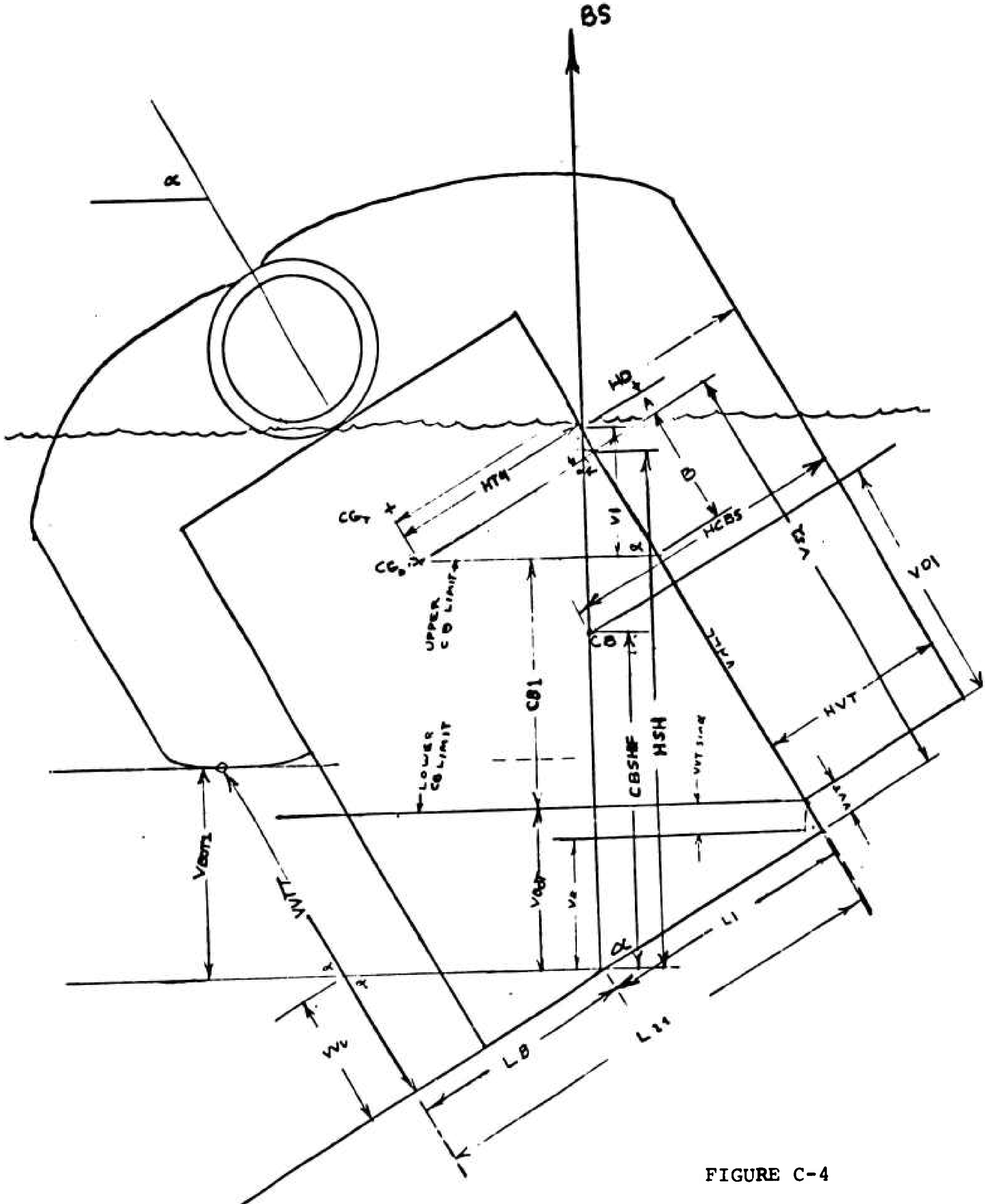


FIGURE C-4

SCHEMATIC FOR DETERMINATION OF VERTICAL LOCATION OF CB OF PARTIALLY EMERGED PFD (A > 0)

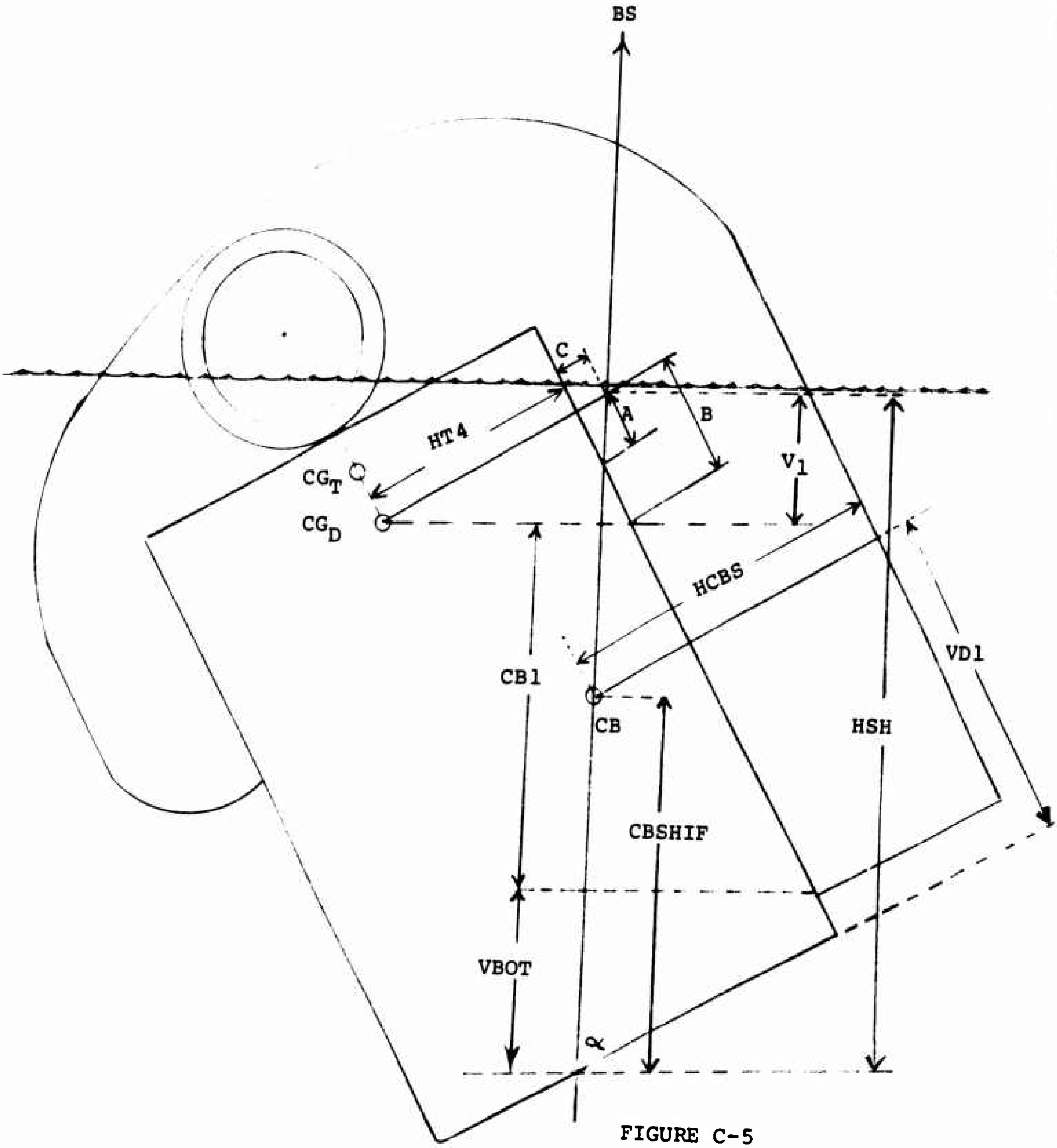


FIGURE C-5

SCHEMATIC FOR DETERMINATION OF VERTICAL LOCATION OF CB OF PARTIALLY EMERGED PFD (A < 0)

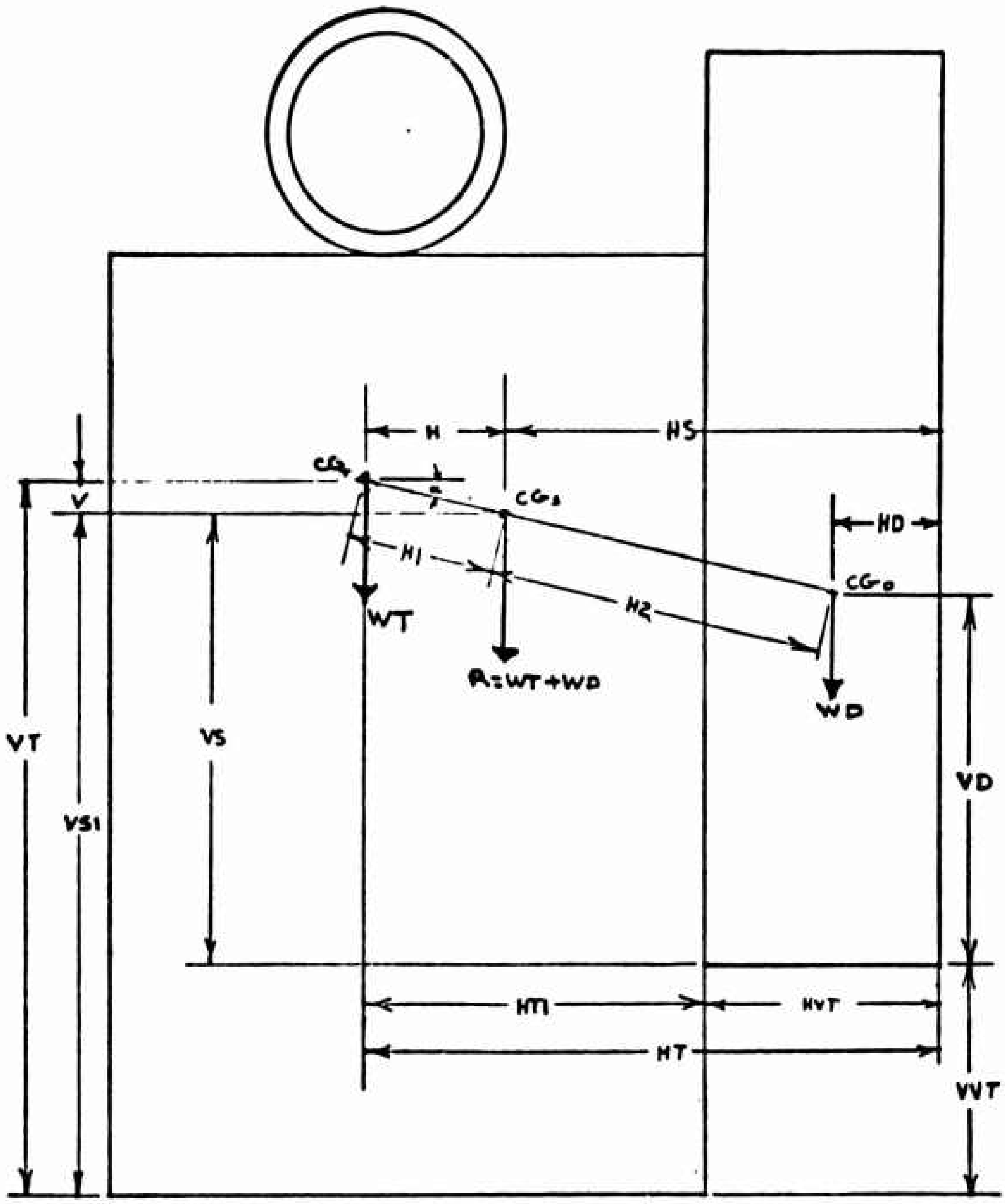


FIGURE C-6

SCHEMATIC FOR DETERMINING CG OF BUOYANT DEVICE

The following pages contain the computer program listings in FORTRAN IV.

ULCB

```
********CB OF PARTIALLY EMERGED PFD**M.R.SUCHOMEL-10/9/73
* ALL DISTANCE IN INCHES, WTS IN LBS, AND ANGLES IN DEG
* HT1=HOR DIST FR FRONT OF TORSO TO TORSO CG
* VT=VERT DIST FR BOTTOM OF TORSO TO TORSO CG
* WT=WT OF TORSO FRAME
* WDD=WT OF DRY SAMPLE DEVICE
 * HVT=HOR DIST FR FRONT OF VEST TO FRONT OF TORSO
 * VVT=VERT DIST FR BOTTOM OF TORSO TO BOTTOM OF VEST
 * HC=HOR DIST FR FRONT OF TORSO TO PT OF W2 ATTACHMENT
 * VC=VERT DIST FR BOTTOM OF TORSO TO PT OF W2 ATTACHMENT
 * HS=HOR DIST FR FRONT OF VEST TO SYSTEM CG
 * VS1=VERT DIST FR BOTTOM OF TORSO TO CG OF SYSTEM
 * BVT=BUOYANCY OF TOTALLY SUBMERGED DEVICE
 * WT11=WT OF TORSO FRAME PARTIALLY EMERGED
 * WTS=WT OF TORSO FULLY SUBMERGED
 * ALPHA=ANGLE FR HORIZ TO TORSO AXIS (90-OBS ANGLE OF SUBJ)
 * W1=WT AT TORSO FRONT TO GIVE ALPHA
 * W2=WT AT TORSO ARM TO GIVE ALPHA
 * VWT=EST. VERT DIST FR TORSO BOT TO CG OF ABOVE-WATER TORSO
 * HWT=EST HOR DIST FR TORSO FRONT TO CG OF ABOVE-WATER TORSO
*L11=HOR DIST FR TORSO FRONT TO LOWEST POINT OF BACK
*    FLOTATION MAT'L--IF NO MAT'L ON BACK, SET L11&VVTT = 0
*VVTT=VERT DIST FR TORSO BOT TO LOWEST POINT OF BACK
* TO OMIT PRINTING CG COORDINATES, ENTER ",1" AT END OF DATA
*
 REAL L,L1,LL
 COMMONHT11,VT11,VD,HD,VS
*TEMPORARILY TO AVOID INSERTING REPETITIVE DATA,
 DATAHT1/4.9/,VT/11.15/,WT/5.19/,HC/7.1/,VC/14.8/,WTS/4.3/
 888CONTINUE
 N=0
 READ,WDD,HVT,VVT,HS,VS1,BVT,WT11,
&ALPHA,W1,W2,VWT,HWT,L11,VVTT,N
 W=WT11
 BS=WT11+W1+W2
 CALLCGMOVE(HT1,HVT,VT,VVT,WT11,VWT,HWT,WTS,WT)
*SUBPROG FOR CALCULATING RESULTANT CG OF ABOVE-&BELOW-WATER
*TORSO,IE SOLVES FOR HT11,VT11,HOR&VERT COORDINATES
 ALPHA=ALPHA/57.2957
 SINA=SIN(ALPHA)
 COSA=COS(ALPHA)
 TANA=TAN(ALPHA)
 H1=HC*SINA
 H2=VC*COS(ALPHA)
 L=(HT11+VT11/TANA)*SINA
 C=H1+H2-L
 AB=(W*C+W1*(H1+H2))/BS
 BC=H1+H2-AB
 L1=BC/SINA
 CALL CGPFD(HT1,HVT,VS1,VVT,VT,HS,WT,WDD)
```

ULCB CONTINUED

```
*CALCULATES HD,VD,RESP HOR&VERT DISTANCE TO DEVICE CG
*UPPER CB LIMIT
 IF(ALPHA.NE.0)GOTO881
 CB1=HVT-HD
 VBOT=0.
 GOTO188
 881CONTINUE
 VS2=VD+VVT
 HT4=HD-HVT
 HSH=VS2/SINA
 A=L1*TANA-VS2
 B=HT4/TANA
*LOWER CB LIMIT
 IF(A)13,11,11
 11 V1=(A+B)*SINA
 GOTO17
 13 C=ABS(A/TANA)
 V1=(HT4+C)*COSA
 17 V2=L1*COSA
 VBOT=V2+VVT*SINA
*FOLLOWING 5 STEPS CHECK FOR A LOWER BUOYANT MAT'L PT ON BACK
 IF(L11.EQ.0.)GOTO101
 LB=L11-L1
 VVV=LB/TANA
 VBOT1=(VVTT-VVV)*SINA
 IF(VBOT1.LT.VBOT)VBOT=VBOT1
 101CONTINUE
 IF(A)21,22,22
 21 CB1=HSH-V1-VBOT
 GOTO188
 22 CB1=L1/COSA-V1-VBOT
 188CONTINUE
 CBSHIF=CB1*BS/(WDD+BVT)+CB1-VBOT-CB1*BVT/(BVT+WDD)
 HCBS=HVT+L1-CBSHIF*COSA
 IF(ALPHA.EQ.0.)HCBS=HVT-CBSHIF
 VD1=CBSHIF*SINA-VVT
 IF(ALPHA.EQ.0.)VD1=VD
 PCNT=BS/BVT*100.
 PRINT77,BS,PCNT,HCBS,VD1
 77 FORMAT("EFFECTIVE BUOYANCY OF PFD(LBS) = ",F6.2,
&" (",F5.1,"% OF FULL VALUE)"/
&"HOR DIST FRO FRONT OF PFD TO CB = ",F6.2/
&"VERT DIST FRO BOTTOM OF PFD TO CB = ",F6.2)
 IF(N.EQ.1)GOTO888
 PRINT35,HD,VD
 35FORMAT("HOR DIST, IN., FR FRONT OF DEVICE TO DEVICE CG = ",
&F6.2/"VERT DIST FR BOTTOM OF DEVICE TO DEVICE CG = ",F6.2)
 GOTO888
  STOP
  END
```

ULCB CONTINUED

```
*
 SUBROUTINECGMOVE(HT1,HVT,VT,VVT,WT11,VWT,HWT,WTS,WT)
 COMMON HT11,VT11
* CALCULATES APPROX LOCATION OF RESULTANT CG AS AFFECTED BY CG
* OF ANY ABOVE-WATER TORSO(HT11&VT11)
 IF(HWT)4,4,5
 4VT11=VT
 HT11=HT1
 RETURN
 5HT=HT1+HVT
 VTT=VT-VVT
 V=VWT-VT
 H=HT1-HWT
 ANG=ABS(ATAN(V/H))
 HTOT=SQRT(V**2+H**2)
* THE ABOVE-WATER WT OF TORSO IS,
 ABVWTR=WT*(WT11-WTS)/(WT-WTS)
* THE UNDERWATER WT OF TORSO IS,
 WTUNDR=WTS*(WT-ABVWTR)/WT
* FROM WTUNDR*H1=ABVWTR*(HTOT-H1)=HTOT*ABVWTR-H1*ABVWTR
 H1=HTOT*ABVWTR/(WTUNDR+ABVWTR)
 85IF(H)20,10,10
 10HT11=HT1-H1*COS(ANG)
 GOTO30
 20HT11=HT1+H1*COS(ANG)
 30CONTINUE
 IF(V)60,40,40
 40VT11=VT+H1*SIN(ANG)
 RETURN
 60VT11=VT-H1*SIN(ANG)
 RETURN
*ALLOWING FOR SHIFT DUE TO ABOVE-WATER TORSO WT ACTING ALONE.
 END
*
 FUNCTION TAN(A)
 TAN=SIN(A)/COS(A)
 RETURN
 END
*
 SUBROUTINE CGPFD(HT1,HVT,VS1,VVT,VT,HS,WT,WD)
* CALCULATES CG OF PFD
 COMMONX,Y,VD,HD,VS
 HT=HT1+HVT
 VS=VS1-VVT
 V=VS1-VT
 H=HT-HS
 H1=SQRT(V**2+H**2)
 ANG=ABS(ATAN(V/H))
 H2=WT/WD*H1
 IF(V)6,4,4
```

ULCB CONTINUED

```
      4VD=VS+H2*SIN(ANG)
      GOTO8
      6VD=VS-H2*SIN(ANG)
      8CONTINUE
     * WHERE VD IS VERT DIST TO DEVICE CG FROM BOTTOM OF DEVICE
     * FOLLOWING ROUTES TO 10 OR 20 DEPENDING ON HOR LOCATION OF
     * CGS RELATIVE TO CGT
      IF(H)20,10,10
      10HD=HS-H2*COS(ANG)
      RETURN
      20HD=HS+H2*COS(ANG)
     * WHERE HD IS HOR DIST FRO DEVICE FRONT TO DEVICE CG
      RETURN
      END
```

ULBV

```
   * ULBV--COMPUTES BUOYANCY VECTORS, SPECIFYING LOCATION
    *       OUT FROM CHEST OR BACK AND UP FROM SSN
      RAD = 57.2958
    333CONTINUE
    READ,E,F,DL2,TL2,DL1,TL1
   *E=CHEST TH.,F=SSN-CVL, DL2&TL2=DIST,ANGL CVL-EAR, HEAD FORW,
   *DL2,TL2=DIST,DEG CVL-EAR, HEAD FORWARD,
   *DL1,TL1=DIST&ANGLE CVL-EAR, HEAD BACK
    3CONTINUE
    NTEST=0
     READ,T1,Y22,B2,Y1,X1,B4,Y3,X3,NTEST
    * WHERE T1=PFD THICK, Y22=SSN TO PFD BOT, B2=EFF. BUOYANCY
    * OF PFD,Y1,X1=Y&X COORDINATES OF CB IN FORWARD
    * POSITION, B4,Y3,X3=RESP VALUES IN BACK POS, NTEST=1 IF NEXT
    * READING TO INCLUDE NEW SUBJECT
     XF=T1-X1
     YF= -(Y22-Y1)
     101FORMAT(1X," MAGF  ANGF  MAGB  ANGB"/1X,4F6.1/)
     BF = B2
     30 X2 = E/2. + XF + DL2*SIN(TL2/RAD)
     Y2 = -DL2*COS(TL2/RAD) + F + YF
     TB2 = ATAN2(ABS(X2),ABS(Y2))*RAD
     CALL QUAD(TB2,X2,Y2)
     AB2 = BF*SQRT(X2**2 + Y2**2)
     XF1=T1-X3
     YF1=-(Y22-Y3)
```

ULBV CONTINUED

```
      BX1 = E/2. + XF1 + DL1*SIN(TL1/RAD)
      BY1 = -DL1*COS(TL1/RAD) + F + YF1
      TB1 = ATAN2(ABS(BX1),ABS(BY1))*RAD
      CALL QUAD(TB1,BX1,BY1)
      AB1=SQRT(BX1**2+BY1**2)*B4
      PRINT101,AB2,TB2,AB1,TB1
      IF(NTEST.EQ.1)GOTO333
      GO TO 3
    2 STOP
      END
*
      SUBROUTINE QUAD(BE,X,Y)
      IF(Y.LT.0.0) GO TO 8
*   Y+
      IF(X.LT.0.0) GO TO 11
*   X+,Y+
      THR = 180. - BE
      GO TO 7
*   X-,Y+
   11 THR = BE - 180.
      GO TO 7
*   Y-
    8 IF(X.LT.0.0) GO TO 9
*   X+,Y-
      THR = BE
      GO TO 7
*   X-,Y-
    9 THR = -BE
    7 CONTINUE
      BE = THR
      RETURN
      END
```

ULSTAB

```
**********ULSTAB*********
*
* COMPILE WITH X TO SUPPRESS PLOT
*
     INTEGER OVERFL,NB2,ND,NA2
      DIMENSION X(2,50),Y(2,50),NPT(2)
      DIMENSION E(2),TH(2),RMSX(2),RMSY(2)
      DIMENSION WX(2,50),AA(2)
```

ULSTAB CONTINUED

```
      DATA AA/"BACK","FWD."/
      111CONTINUE
      IL = 0
      RAD = 57.2958
*     READ IN RAW DATA AND CORRECT ANGLES
    1 DO 15 I=1,2
        DO 15 J=1,50
      WX(I,J) = 0.0
       X(I,J) = 0.0
   15 Y(I,J) = 0.0
    9 FORMAT(A6)
      PRINT,"ENTER DATA FILE NAME ",
      READ 9,FILENM
      CALLOPENS(MM,1,FILENM,2)
      IF(MM) 8,7,8
    8 PRINT,"INPUT FILE NOT AVAILABLE","",""
      GO TO 9
    7 CONTINUE
   16 READ(1,,END=17)NS,NA
      IL = IL + 1
  110 FORMAT("LINE",I6)
  105 FORMAT(2I5)
      NF = 1
   13 J = 0
  920 READ(1,,END=17)W,NB2
      IL = IL + 1
  100 FORMAT(F10.2,I5)
  910 CONTINUE
      READ(1,,END=17)ND,NA2
      IL = IL + 1
  101 FORMAT(2I5)
      IF(ND.LT.-99) GO TO 920
      IF(ND.GT.99) GO TO 20
      J = J + 1
      X(NF,J) = ND
      Y(NF,J) = (2.0/RAD)*(NA2 - NB2)
      WX(NF,J) = W
      GO TO 910
   20 CONTINUE
      NP = J
      NPT(NF) = NP
* CONVERT RAW DATA TO ANGLE-TORQUE
* ALL ANGLES IN RADIANS EXCEPT FOR OUTPUT
      PRINT,"      DEG. IN-LBS"
      DO 50 I = 1,NP
      TEMP = WX(NF,I)*X(NF,I)*COS(Y(NF,I)+2*NB2/RAD)
      X(NF,I) = Y(NF,I)
      XD = X(NF,I)*RAD
      Y(NF,I) = TEMP
      PRINT 201,I,XD,Y(NF,I)
```

```
   50 CONTINUE
   GO TO (11,12),NF
   11 NF = 2
   GO TO 13
   201 FORMAT(I3,2F6.1)
   104 FORMAT(I5)
 *
 *  DO LSQ SINE FIT
   12 DO 39 NF = 1,2
   3 D11 = 0.0
    D12 = 0.0
    D22 = 0.0
    C1 = 0.0
    C2 = 0.0
   STH2 = 0.0
   STH = 0.0
   SY2 =0.0
   SY=0.0
   SYTH = 0.0
   NP = NPT(NF)
   DO 30 I = 1,NP
   PH = X(NF,I)
   ST = SIN(PH)
   CT = COS(PH)
   D11 = D11 + ST**2
   D12 = D12 + ST*CT
   D22 = D22 + CT**2
   C1 = C1 + ST*Y(NF,I)
   C2 = C2 + CT*Y(NF,I)
   STH2 = STH2 + PH**2
   STH = STH + PH
   SY2 = SY2 + Y(NF,I)**2
   SY = SY + Y(NF,I)
   SYTH = SYTH + PH*Y(NF,I)
   30 CONTINUE
   D = D11*D22 - D12**2
  A = (C1*D22 - C2*D12)/D
   B = (D11*C2 - D12*C1)/D
   TH(NF) = -ATAN2(B,A)
   E(NF) = SQRT(A**2 + B**2)
 *  COMPUTE RESIDUAL OF ERRORS IN FIT
 *  AUXILLARY SUMS
   PN = NP
   DSTH2 = STH2 - STH**2/N
   DSYTH = SYTH - SY*STH/PN
   SIG2E = (DSTH2*(SY2-SY*SY/PN)-DSYTH**2)*DSTH2**(-2)/(PN-2.0)
   SIG2TH = SIG2E*((SY/PN)**2)*(DSTH2**3)*(DSYTH**(-2))*
  & (PN*(SY**(-2)) + DSTH2*(DSYTH**(-2)))
   RMSY(NF) = SQRT(SIG2E)
   RMSX(NF) = SQRT(SIG2TH)*RAD
```

ULSTAB CONTINUED

```
      THD = TH(NF)*RAD
      PRINT,"INTRINSIC STIFFNESS VECTORS","
      PRINT 302,NS,AA(NF)
      PRINT 200,E(NF),THD
      302 FORMAT("  SUBJECT",I4," ",A4)
      200 FORMAT(/,"  A      =",F10.2," IN-LBS",
     & /,"   THETA =",F10.2," DEGREES")
      300 FORMAT("  STD. DEV. IN A     IS",F10.3,/,
     & "  STD.DEV. IN THETA IS",F10.3,//)
      39 CONTINUE
      THD1 = TH(1)*RAD
      THD2 = TH(2)*RAD
     * PRINT INTRINSIC STIFFNESS VECTORS IN FILE
      GO TO 704
      PRINT 333,NS,NA, E(1),THD1,E(2),THD2
      333FORMAT("NS,NA,E(1),THD1,E(2)&THD2=",I4,I3,4F6.1," RESP.")
      704 CONTINUE
      301 FORMAT("RMSX(1)=",F6.1,
     &/"  RMSY(1)=",F6.1,", RMSX(2)=",F6.1,", RMSY(2)=",F6.1)
      PRINT 301,RMSX(1),RMSY(1),RMSX(2),RMSY(2)
     *
      CALLCLOSEF(1)
      GOTO111
      17 STOP17
      END
```

ULADV

```
   *********** PROGRAM ADV **********
   ** ADDS TWO VECTORS IN POLAR FORM
   **
    DIMENSION A(2),TH(2),AL(2),THL(2)
    DIMENSION AR(2),THRD(2)
    RAD = 57.2958
    PI = 3.14159
    PRINT111
    1 READ, NS,A(1),TH(1),A(2),TH(2)
    PRINT102
    READ, NSL,AL(1),THL(1),AL(2),THL(2)
    111FORMAT("WHAT ARE NS,A(1),TH(1),A(2),TH(2)")
    102FORMAT("WHAT ARE NSL,AL(1),THL(1),AL(2),THL(2)"/
   &    "(NOTE THAT NSL MUST = NS)")
    IF(NS.NE.NSL) GO TO 999
```

ULADV CONTINUED

```
DO 10 K=1,2
T1 = TH(K)/RAD
T2=THL(K)/RAD
XP = A(K)*SIN(T1)
YP = A(K)*COS(T1)
X = XP + AL(K)*SIN(T2)
Y = YP + AL(K)*COS(T2)
AR(K) = SQRT(X**2 + Y**2)
BE=ATAN(ABS(X/Y))
IF(Y.LT.0.00) GO TO 8
IF(X.LT.0.00) GO TO 11
THR = BE
GO TO 7
11 THR = -BE
GO TO 7
8 IF(X.LT.0.00) GO TO 9
THR = PI - BE
GO TO 7
9 THR = -PI + BE
7 CONTINUE
THRD(K) = THR*RAD
10CONTINUE
PRINT101,NS,AR(1),THRD(1),AR(2),THRD(2)
101 FORMAT("NS=",I4,", AR(1)=",F6.1,"
& , THRD(1)=",F6.1,/" AR(2)=",F6.2,", THRD(2)="F6.1)
GOTO1
5 STOP 5
999 STOP 999
END
```

APPENDIX D

Sensitivity Analysis of Flotation Theory

The condition of equilibrium of a person and buoyancy device is given by Eq. IV-1 of Reference 1, that is:

$$M(\theta) = Wd_T \sin(\theta - \theta_T) + \rho_W \Delta V_L d_L \sin(\theta - \theta_L) - Bd_B \sin(\theta - \theta_B) = 0 \qquad \text{(D-1)}$$

For this analysis, the dependences of d_T, B, and d_B on the angle θ are neglected. For simplicity, define the following:

$I = Wd_T$ = magnitude of the intrinsic stiffness vector

$L = \rho_W \Delta V_L d_L$ = magnitude of the lung volume vector

$\beta = -Bd_B$ = negative magnitude of the buoyancy device vector

In general, we have the functional relationship

$$\theta = f(I, L, \beta, \theta_T, \theta_L, \theta_B) \qquad \text{(D - 2)}$$

and we wish to determine the changes in θ that would be produced by small changes in the independent variables. This may be expressed through the total differential of θ as follows:

$$d\theta = f_I dI + f_L dL + f_\beta d\beta + f_{\theta_T} d\theta_T + f_{\theta_L} d\theta_L + f_{\theta_B} d\theta_B$$

where: $f_I = \partial f/\partial I$, $f_L = \partial f/\partial L$, etc. (D - 3)

$f_I dI$, $f_L dL$, and $f_\beta d\beta$ are in radians.

To evaluate these partial derivatives, it is most convenient to implicitly differentiate equation (D-1). This yields the following:

$$f_I = \partial\theta/\partial I = -\frac{\partial M/\partial I}{\partial M/\partial\theta} \quad \text{(D-4)}$$

$$f_L = \partial\theta/\partial L = -\frac{\partial M/\partial L}{\partial M/\partial\theta} \quad \text{(D-5)}$$

$$f_B = \partial\theta/\partial B = -\frac{\partial M/\partial B}{\partial M/\partial\theta} \quad \text{(D-6)}$$

$$f_{\theta_T} = \partial\theta/\partial\theta_T = -\frac{\partial M/\partial\theta_T}{\partial M/\partial\theta} \quad \text{(D-7)}$$

$$f_{\theta_L} = \partial\theta/\partial\theta_L = -\frac{\partial M/\partial\theta_L}{\partial M/\partial\theta} \quad \text{(D-8)}$$

$$f_{\theta_B} = \partial\theta/\partial\theta_B = -\frac{\partial M/\partial\theta_B}{\partial M/\partial\theta} \quad \text{(D-9)}$$

By differentation:

$$\partial M/\partial I = \sin(\theta - \theta_T) \quad \text{(D-10)}$$

$$\partial M/\partial L = \sin(\theta - \theta_L) \quad \text{(D-11)}$$

$$\partial M/\partial B = \sin(\theta - \theta_B) \quad \text{(D-12)}$$

$$\partial M/\partial\theta_T = -I\cos(\theta - \theta_T) \quad \text{(D-13)}$$

$$\partial M/\partial\theta_L = -L\cos(\theta - \theta_L) \quad \text{(D-14)}$$

$$\partial M/\partial\theta_B = -B\cos(\theta - \theta_B) \quad \text{(D-15)}$$

$$\partial M/\partial\theta = I\cos(\theta - \theta_T) + L\cos(\theta - \theta_L) + \beta\cos(\theta - \theta_B) \quad \text{(D-16)}$$

With these relations, each of the partial derivatives (f_I, f_L, f_B, etc.) in Eq D-3 can be calculated for a particular case, and the sensitivity of θ to small changes in each of the other variables evaluated. It may be noted that the partial derivatives may be very large if $\partial M/\partial\theta$ is very small. The value of $\partial M/\partial\theta$ is the vertical component of the resultant stiffness vector, and is equal to the magnitude of that vector at equilibrium (the horizontal component is the turning moment, which is zero at equilibrium). Thus the condition of small $\partial M/\partial\theta$ corresponds to the situation of small resultant stiffness vector, when the equilibrium is not very stable. It is not surprising therefore, that small changes in the variables would lead to large changes in θ under those conditions.

The sensitivity of θ to experimental errors in the various vector quantities is illustrated below for two cases cited in Reference 1 and summarized in Figs. D-1 and D-2 (which are reproduced from Figs. A-3 and A-4 of Reference 1). The results are:

	Fig. D-1 RSV = 41	Fig. D-2 RSV = 76
Error in θ for 1° error in θ_T	2.2°	0.65°
Error in θ for 1° error in θ_L	0.47°	0.15°
Error in θ for 1° error in θ_β	1.6°	0.19°
Error in θ for 1 percent error in I	0.72°	0.81°
Error in θ for 1 percent error in L	0.74°	0.45°
Error in θ for 1 percent error in β	1.4°	1.3°

Since the value of the resultant stiffness vector $\partial M/\partial \theta$ is not particularly small for either of these cases, neither represents an extreme situation. Yet the possible errors induced in the calculated values of θ could be quite large. For example, if experimental errors of 10 percent in I, L and β produced additive errors in θ, the resultant error in θ would be 28 degrees for Fig. D-1 and 25 degrees for Fig. D-2.

A more extreme situation is represented by the calculation for Subject 60 of Reference 1. It has been assumed here that the buoyancy device and its buoyancy vector are the same as that for the Fig. D-2 situation. The result of the sensitivity analysis is:

	Subject 60 RSV = 13
Error in θ for 1 degree error in θ_T	0.43°
Error in θ for 1 degree error in θ_L	1.68°
Error in θ for 1 degree error in θ_β	1.11°
Error in θ for 1 percent error in I	3.46°
Error in θ for 1 percent error in L	2.12°
Error in θ for 1 percent error in β	5.65°

For this case, where the magnitude of the resultant stiffness vector is small (13), errors in I, L, and β have a large effect on the accuracy of the calculated

value of θ. For example, if experimental errors of 10 percent in I, L, and ε produced additive errors in θ, the resultant error in θ would be 111 degrees, according to the sensitivity analysis. It is noted that the use of Eq D-3 to calculate finite changes in θ for finite changes in the other variables is accurate only for small increments. Thus the large increments in θ which are calculated here are not exact, but indicate only the potential for large errors which exists.

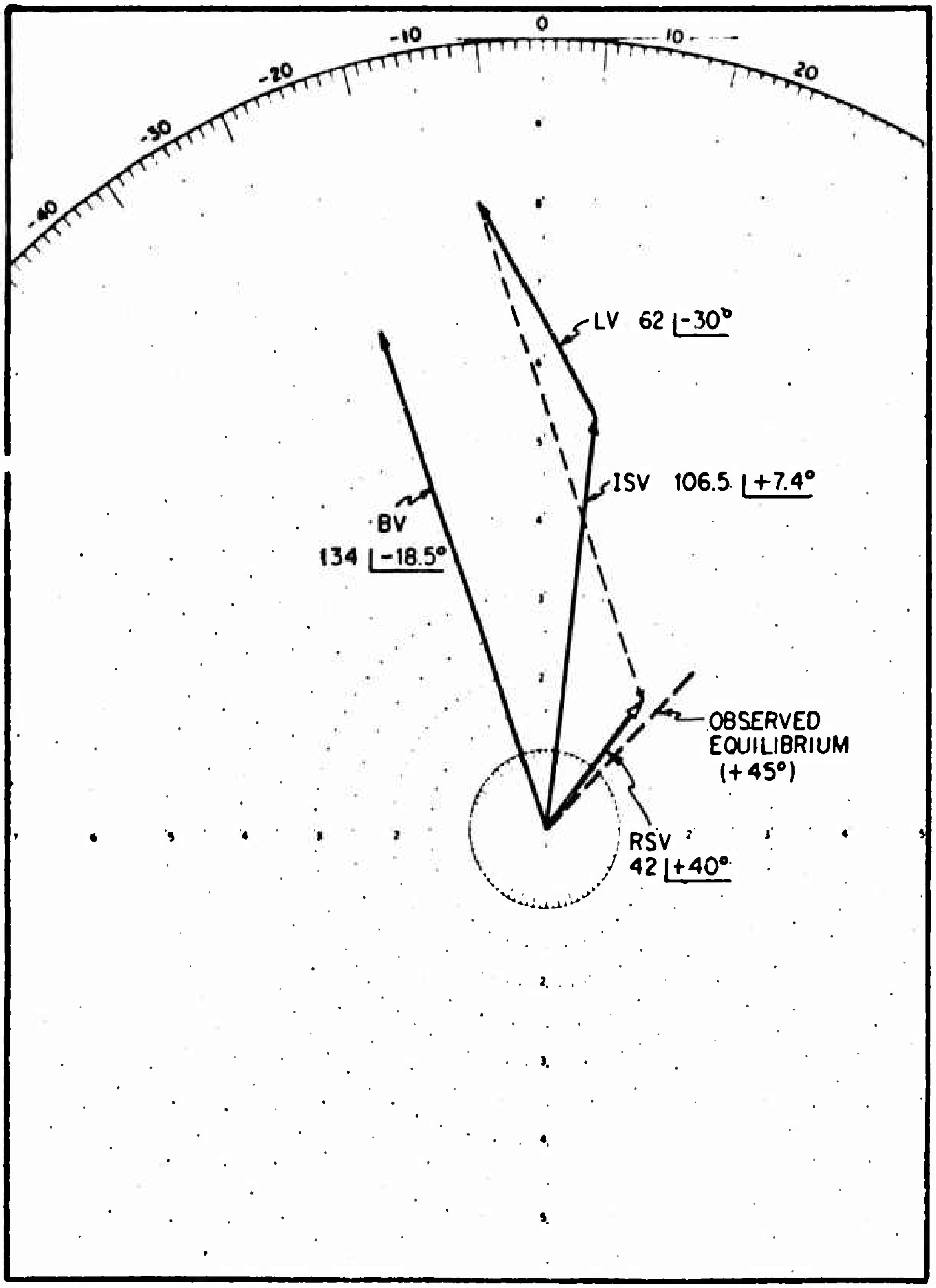


FIGURE D-1
Determination of Resultant Stability Vector - Head Forward

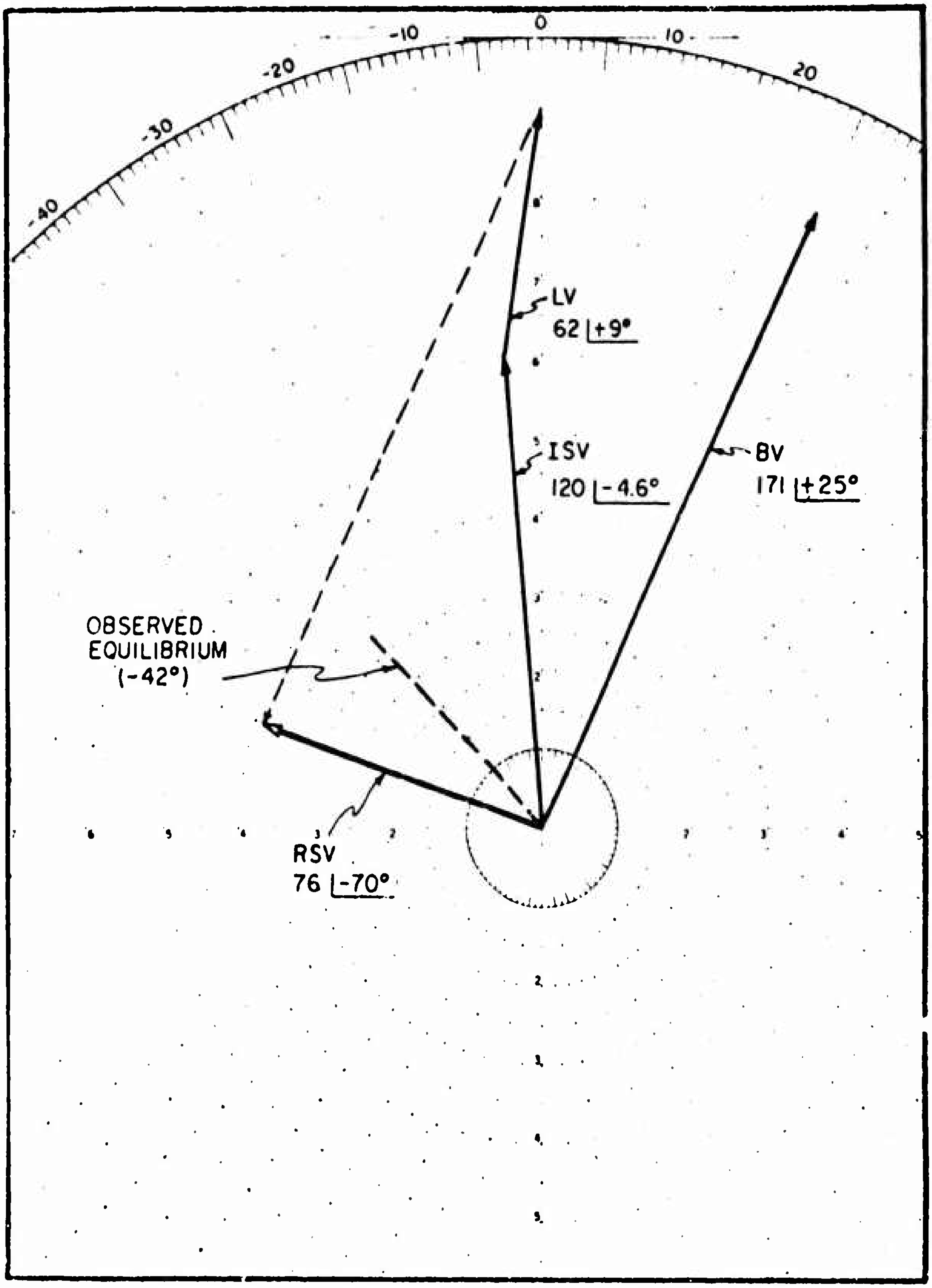


FIGURE D-2

Determination of Resultant Stability Vector - Head Back